Entrance Syllabus

for Ph.D. in Mathematics / Applied Mathematics, 2023 - 24

Note for paper setting: * The question paper shall contain 70 Objective type questions (Multiple choice, each with four choices) uniformly distributed over the entire syllabus.

* Each question will be of One mark.

UNIT		
1	Elementary Set theory & Real Number System	Finite, countable and uncountable sets. Real number system as a complete ordered field. Archimedean property of real numbers. Various properties of supremum and infimum. Cauchy-Schwarz inequality. Extended Real number system.
2	Sequences and series of numbers	Bounded, monotonic, convergent, divergent and Cauchy sequences. limsup & liminf of a sequence of numbers. Convergence of series of numbers and various convergence tests. Absolute convergence.
3	Continuity of Real functions	Limit & Continuity of a function and their elementary properties. Fundamental theorems - Intermediate value theorem, boundedness of continuous functions on closed intervals etc. Uniform continuity and its connection with continuity. Various types of Discontinuities.
4	Differentiability of real functions	Definition, examples and basic properties of differentiable functions. Rolle's & Lagrange's Mean Value theorem. Taylor's series. Intermediate value theorem for derivatives. Relation of maxima and minima of a function with derivative. Convex functions. Monotonic functions.
5	Sequences and Series of functions	Sequences and Series of functions - pointwise and uniform convergence. Connection of Uniform convergence with Continuity, Differentiation & Riemann Integration. Existence of a continuous and nowhere differentiable function & Weirstrss approximation theorem.
6	Riemann Integration	Riemann integral and various sufficient conditions for the existence of Riemann integral. Elementary properties of Riemann Integral. Fundamental theorem of Calculus.
7	Functions of bounded variation & Improper Riemann Integral	Functions of bounded variation with various operations on them. Total Variation. Improper Integrals - Convergence, Absolute convergence & various comparison Tests.

UNIT		
8	Lebesgue Integration of real functions - I	Sigma Algebra of a sets- Borel Sigma Algebra. Measure on a Sigma Algebra. Lebesgue outer measure on R. Lebesgue measureable sets. Measureable functions and various operations on them.
9	Lebesgue Integration of real functions- II	Lebesgue Integral and its relation with Riemann integral. Monotone convergence theorem & Fatou's lemma. Essential supremum of a function. LP spaces - Holder's, Minkowski & Jensen's inequality. Convergence in LP spaces.
10	Functions of Several (Real) Variables	Directional & partial derivatives. Total Derivative (Derivative as a Linear transformation) and its connection with continuity. The Jacobian matrix. Chain rule. Mean value theorem. Connection between total & Partial derivatives. Inverse and implicit function theorems.
11	Metric Spaces -I (Fundamental)	Definition and standard examples of metric spaces. Open and closed sets. Closure and interior of a set. Cauchy sequences. Convergent sequences & their relation with closure. Complete metric spaces. Baire's category theorem. Dense sets & Separable metric spaces.
12	Metric Spaces -II (Continuity)	Continuity of a function & its relation with inverse images of open / closed sets, convergent sequences & closure of a set. The space C[a, b] as a vector and complete metric space. Contraction mapping & fixed point theorem.
13	Metric Spaces -III (Compactness)	Compact sets, their continuous images and their relation with closed and bounded sets. Relation of a compactness of a metric space with Bolozano- Weirstrass property, sequential compactness, Countable compactness & Total boundedness plus completeness.
14	Metric Spaces -IV (Fundamental theorems)	Bolzano - weirstrass, Cantor's intersection theorem, Lindeloff covering theorem and Heine Borel theorem for the Euclidean space Rn.
15	Metric Spaces -V (Connectedness)	Equicontinuous family of functions. Ascoli theorem. Boundedness of a continuous function with compact domain. Continuous functions with compact domain. Connected set and its continuous image. Generel version of Intermediate value theorem. Pathwise connected sets and their relation with connected sets.
16	Normed Linear Spaces	Norm on a vector space with standard examples. Equivalence & completeness of norms on FD spaces. Compactness of B[0, 1]. Completeness and absolute convergence.
17	Inner Product Spaces	Inner product with elementary properties. Orthogonal complement of a set. Projection theorem. Orthonormal & Total orthonormal sets. Various conditions for a set to be a Total orthonormal set. Separability and orthonormal sets.
18	Bounded Linear Operators	Bounded linear operators. Boundedness of LOs on FD spaces. Completeness of the normed space of operators. Dual spaces of Euclidean and little lp spaces. Compact operators. Self adjoint bounded linear operators between Hilbert spaces.

UNIT		
19	Fundamental thorems on Normed and Hilbert spaces	Hahn Banach extension theorem and its consequences. Baire's category theorem. Uniform boundedness theorem with applications. Open mapping theorem. Closed graph theorem. Riesz representation theorem.
20	Spectral theory of BLOs	Spectrum and resolvant of a BLO. Non-emptiness, closedness and boundness of the spectrum of a BLO. Spectral mapping theorem for polynomials. Spectral radius. Spectrum of a Self adjoint Bounded linear operator.
21	Vector Spaces	Vector spaces with standard examples. Linearly Independent sets. Span of a set. Basis and dimension of a vector space. Linear transformations with elemntary results.
22	Matrices & Determinants	Algebra of matrices. Rank and determinant of a matrix. Eigenvalues and eigenvectors. Cayley-Hamilton theorem. Quadratic forms. Reduction and classification of quadratic forms.
23	Matrix representation of linear transformations	Matrix representation of a linear transformation. Change of basis. Canonical forms. Diagonal forms. Triangular forms. Jordan forms.
24	Algebra - I (Groups)	Groups, subgroups, normal subgroups & quotient groups. Homomorphism & isomorphism. Finite, infinite and cyclic groups. Permutation group and Cayley's theorem.
25	Algebra - II (Cauchy and Sylow's theorems)	Conjugate of an element of a group, class equation and its applications. Cauchy's and Syllow's theorems. Applications of Syllow's theorem in the determination of simplicity of groups.
26	Algebra - III (Rings, ED, UFD, PID)	Rings and its special classes. Characteristic of an integral domain. Homomorphism and isomorphism. Ideals and quotient rings. Euclidean domain. Proncipal ideal domain, Unique factorization domain. The ring of polynomials R[x]. Polynomials in Q[x] - primitive polynomials, content of a polynomial, Gauss lemma & Einstein's criteria.
27	Algebra - IV (Fields)	Classification, structure and subfields of a finite field. Field extensions - finite, infinite, algebraic & splitting.
28	Complex Analysis - I (Fundamentals)	Complex numbers as ordered pairs of real numbers. Algebra of complex numbers, Stereographic projection, Polar form of a complex number, Impossibility of ordering of complex numbers, Complex polynomials, Power series, Transcendental functions such as exponential, trigonometric and hyperbolic functions.
29	Complex Analysis -II (Analytic functions)	Limit, continuity & Differentiability of a complex function. Analytic and harmonic functions. Conformal mappings with elementary examples and properties. Mobious Transformations with basic properties.

UNIT		
30	Complex Analysis -III (Integration)	Contour integral and its basic properties. Cauchy theorem & Cauchy's integral formula. Morera's theorem. Cauchy's inequality. Liouville's theorem. Fundamental theorem of Algebra. Convex Hull. Gauss & Luca's theorem. Gauss mean value property. Max./Min. Modulus principle. Schwarz lemma & Open mapping theorem
31	Complex Analysis -IV (Singularities & Residues)	Taylor's and Laurents's theorem. Removable singularities, Poles and Essential singularities. Residues and Cauchy Residue theorem. Argument principle. Casorati-Weirstrass theorem. Rouche's theorem. Evaluation of Definite integrals.
32	Ordinary Differential Equations	Existence and uniqueness of solutions of IVP for first order ODE. Singular solutions of first order ODEs, System of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs. Variation of parameters. Sturm-Liouville BVP. Green's function.
33	Partial Differential Equations	Lagrange and Charpit methods for solving first order PDEs. Cauchy problem for first order PDEs. Classification of second order PDEs. General solution of higher order PDEs with constant coefficients. Method of separation of variables for Laplace, Heat and Wave equations.
34	Numerical Methods	Numerical solutions of algebraic equations. Method of iteration and Newton-Raphson method. Rate of convergence. Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods. Finite differences. Lagrange , Hermite and spline interpolation.
35	Numerical Methods for ODE and PDE	Numerical differentiation and integration. Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.
36	Linear Integral Equations	Linear integral equation of the first and second kind of Fredholm and Volterra type. Solutions with separable kernels. Characteristic numbers and eigenfunctions. Resolvent kernel.
37	Scientific Research -I	Meaning, objectives and significance of scientific research. Types of research approaches. Quantitative research methods. Research problems. Research design, its necessity and types. Ethics in research. Literature survey for research work. Formulation of research title.
38	Scientific Research -II	Significance of report writing, Structure and components of research report. Research papers. Thesis and Research Project reports. Precautions for writing research reports. Pictures and Graphs. Citation Styles. Oral presentation.
39	LATEX & MATLAB	Exposure to LaTeX, Installation, MikTeX & TeXnic Center. Creating reports and articles. Text environment. Math environment. Figures, Tables and BibTeX. Camera Ready Preparation. Introduction to MatLab with its basic commands.
40	Probability	Sample space. Probability and conditional probability of an event. Independent events. Baye's theorem. Random variable (discrete and continuous); Expectation, variance, standard deviation and moments of a random variable. Bernoulli trials.

UNIT		
41	C - Language	History of C language. Structure of C program. Variables, constants, keywords, operators and data types in C. Decision making statements. Loops in C. Arrays. Functions call by value & by reference. Recursive functions. Structure and pointer.

Head of the Department