

# **Syllabus**

## **M. Sc. MATHEMATICS**



**BABAGHULAM SHAH BADSHAH UNIVERSITY**  
**Rajouri -185131, (J&K), INDIA**

## **PROGRAMME SPECIFIC OUTCOMES (PSO)**

### **M. Sc. MATHEMATICS**

After completing this program successfully, we expect a student

- PSO1.** to have sufficient understanding of some core areas of Mathematics such as Real and Complex Analysis, Topology, Functional Analysis and Operator theory, Abstract and Linear Algebra, Galois theory, ODE, Numerical Analysis etc.
- PSO2.** to have sufficient knowledge of applications of above mentioned mathematical areas in various other fields of Mathematics and real world.
- PSO3.** to have deep understanding of that part of Mathematics which he or she encounters in National and State Level Eligibility tests(NET/ SET).
- PSO4.** to have a training of surfing internet for research purposes and to do team work through projects undertaken by the students in final semester.
- PSO5.** is prepared with such a strong background of the subject that he/she can do quality research of international repute in core areas of Mathematics.
- PSO6.** is prepared with such a strong background of the subject that he / she can acquire advanced knowledge of the subject independently.
- PSO7.** is enabled with good communication skills in both verbal and written forms.
- PSO8.** gets enough confidence to speak before gatherings by making him to go through a series of presentations in the class room during his / her course of the study.
- PSO9.** have sufficient computer skills which he / she requires in his / her further studies in the subject.
- PSO10.** have sufficient introduction to Mathematical software MATLAB, LATEX and SPSS.

## SEMESTER I

Course Code	Course Title	No. of Credits	Distribution of Marks		
			SA	UE	Total
<b>Core Courses</b>					
MS- 101	Topology and its Applications	04	40	60	100
MS -102	Techniques in Differential Equations	04	40	60	100
MS -103	Real Analysis	04	40	60	100
MS -104	Applied Numerical Analysis	04	40	60	100
MS -105	Computer Fundamentals and C-Programming	04	40	60	100
MS -106	Lab Course on MS-104 and MS-105	04	50	50	100
	<b>Total</b>	<b>24</b>	<b>250</b>	<b>350</b>	<b>600</b>

**SA: Sessional Assessment**

**UE: University Examination**

## SEMESTER - I

**Course Title: Topology and  
its Applications**

**Course Code: MS-101**

**Credits: 4**

**Maximum Marks: 100**

**University Examination: 70**

**Sessional Assessment: 30**

**Duration of Exam: 3 hours**

### Objectives

*The aim of this course is to introduce the students to the basic ideas of metric and topological spaces and make them appreciate their applications.*

### Unit I

**Elements of point set topology in  $\mathbb{R}^n$ :** Norm and its properties; Open and closed sets; structure of open sets in  $\mathbb{R}$ ; accumulation points; Bolzano- Weirstrass theorem; Cantor intersection theorem; Lindelof covering theorem; Heine Borel theorem; compactness in  $\mathbb{R}^n$ .

### Unit II

**Metric spaces:** Definition and examples— $\mathbb{R}^n$ ,  $\mathbb{C}^n$ ,  $l^p$ ; point set topology; compact sets; convergent and Cauchy sequences; complete metric spaces— $\mathbb{R}^n$ ,  $\mathbb{C}^n$ ,  $l^p$ ; continuity of a function; relation between continuity and inverse images of open/closed sets and convergence of sequences; error correcting codes – Hamming distance; DNA sequences.

### Unit III

**Topological spaces:** Definition and examples; Basis of a Topology; Closed sets; Hausdorff space; modeling of digital image displays in digital topology by topological spaces; interior and closure of a set with their basic properties; limit points; applications to geo information system.

### Unit IV

**Sub-spaces, quotient spaces and continuous functions:** Subspace topology; open and closed sets in subspaces; product of two topological spaces – annulus, solid torus; Quotient spaces – Klein bottle, projective plane; open set definition of continuity and its various equivalent conditions; Pasting lemma; Homeomorphism; Homeomorphic image of a Hausdorff space.

### Unit V

**Connectedness and compactness:** Connected, path connected spaces and their continuous images; arbitrary union and finite product of connected spaces; totally disconnected spaces; connectedness of  $\mathbb{R}^n$ ; general version of intermediate theorem; applications to population model; Compact spaces, subspaces and their continuous images; finite union, product and arbitrary intersection of compact spaces; tube lemma.

### Course Outcomes:

On successful completion of this course, we expect that a student

1. Should be able to explain the concepts of Euclidean, Metric & Topological spaces with standard examples.

2. Should be able to explain the concepts and properties of interior & accumulation points, open, closed, connected & compact sets in Euclidean, Metric & Topological spaces.
3. Should be able to explain the concepts of closure & interior of a set in a topological space and their various properties.
4. Should know the fundamental theorems such as Bolzano- Weirstrass theorem(BWT), Cantor's intersection theorem(CIT), Lindelof covering theorem(LCT), Heine - Borel theorem(HBT) in  $\mathbb{R}^n$  and also should be able to explain the validity of these theorems (as usually been stated in  $\mathbb{R}^n$ ) in general metric spaces.
5. Should be able to explain the connection between metric spaces and error correcting codes and DNA sequences.
6. Should be able to explain the connection between topological spaces and modeling of digital image displays and applications to geo information system.
7. Should be able to explain the concept of convergence of a sequence and Cauchy sequence and their various properties in Topological spaces.
8. Should be able to explain the concept of Continuity with its various versions in Topological spaces and its connections with connected and compact sets.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Adams, C. and Franzosa, R. (2009)**, Introduction to Topology – Pure and Applied, **Pearson**.
2. **Apostol, Tom M., (2002)**, Mathematical Analysis, 1<sup>st</sup> edition, **Narosa Publishing House**.

**REFERENCE BOOKS:**

1. **Willard, S., (1976)**, General Topology **(1970)**, **Dover Publications New York..**
2. **Searcoid, M. O., (2007)**, Metric Spaces, **Springer**.
3. **Munkers J.R. ,(2000)**, Topology, 2<sup>nd</sup> Edition, **PHI**.

## SEMESTER I

**Course Title: Techniques in Differential Equations**  
**Course Code: MS -102**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives

*The main objective of this course is to introduce students to the techniques of solving various differential equations.*

### Unit I

**Higher order linear differential equations-I:** Basic existence theorem (Proof not included); basic theorems on linear homogenous equations; concept of Wronskian; reduction of order method; general solution of a homogeneous linear differential equation with constant coefficients.

### Unit II

**Higher order linear differential equations-II:** Method of undetermined coefficients; method of variation of parameters; Cauchy–Euler equation; power series about an ordinary point; singular point; method of Frobenius; Bessel’s equation and Bessel’s functions.

### Unit III

**Systems of linear differential equations:** Types of linear systems; differential operator; operator method for linear systems with constant coefficients; basic theory of linear systems in normal form; matrix method of solving homogenous linear system with constant coefficients.

### Unit IV

**Laplace transform:** Definition, examples and basic properties of Laplace transform; existence of Laplace transform; step function; inverse Laplace transform and convolution theorem; solution of linear differential equations with constant coefficients by using Laplace transform; linear systems.

### Unit V

**Sturm-Liouville boundary value problems:** Sturm-Liouville problems; characteristic values; characteristic functions; orthogonality of characteristic functions; expansion of a function in a series of orthogonal functions; expansion problem; trigonometric Fourier series and its convergence.

### **Course Outcomes:**

After studying this course we expect a student have understood

1. The concept of homogeneous and non-homogeneous linear differential equations and the method of finding its general solution.
2. How to find the power series solution of homogeneous differential equations at singular points and ordinary points.
3. How to find the solution of linear system by operator method.
4. The basic theory of linear system of differential equations in normal form & matrix method for solving homogeneous linear system with constant coefficients.
5. The concept of Laplace transform & its basic properties.
6. How to find the solution of linear differential equation by using Laplace transform.
7. The concept of Sturm-Liouville problem, orthogonality of characteristic functions & expansion of functions in a series of orthogonal functions.
8. The concept of trigonometric Fourier series and its convergence.

### **Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

### **Books Recommended:**

#### **TEXT BOOKS:**

1. **Ross, S., (1984),** Differential Equations, 3<sup>rd</sup> Edition, **Wiley India (P) Ltd, New Delhi.**

#### **REFERENCE BOOKS:**

1. **Boyce, W.E., DiPrima, R.C., (2007),** Elementary Differential Equations and Boundary Value Problem, 8<sup>th</sup> edition, **John Wiley and sons.**
2. **Edward, P., (2005),** Differential Equation and Boundary Value Problems; Computing and Modeling, 3<sup>rd</sup> edition, **Pearson Education.**
3. **Simmons, G. F., (2003),** Differential Equation with Applications and Historical Notes, 2<sup>nd</sup> edition, **Tata McGraw Hill edition.**

## SEMESTER I

**Course Title: Real Analysis**  
**Course Code: MS -103**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### **Objectives**

*The objective of this course is to introduce students to Riemann and Lebesgue Integration, and make them learn the convergence issues of sequences and series of functions.*

### **Unit I**

**Riemann integral:** Upper and lower sums; Riemann integral and basic criterion for its existence; basic properties of Riemann integral; connection of monotonicity and continuity with the existence of Riemann integral; fundamental theorem of calculus.

### **Unit II**

**Sequences and series of functions:** Point-wise and uniform convergence; Cauchy criterion for uniform convergence; Weirstrass M-test; connection of uniform convergence with differentiation, integration and continuity; example of a continuous and nowhere differentiable function; Weirstrass approximation theorem.

### **Unit III**

**Improper integrals and the Lebesgue Measure:** Definition, examples and convergence of improper integrals of type-I and type-II; absolute convergence and comparison tests; Outer measure; outer measure of an interval in  $\mathbb{R}$ ; measurable sets, Lebesgue measure; Borel sets.

### **Unit IV**

**Measurable real functions:** Non-measurable sets; measurable functions and their sum, difference and product; sequence of measurable functions; the concept of almost everywhere; Integral of a simple function; integral of a bounded measurable function; connection between Riemann and Lebesgue integral; bounded convergence theorem.

### **Unit V**

**Lebesgue integration:** Integral of a non-negative function; Fatou's Lemma; monotone convergence theorem; Lebesgue integral of general function; Lebesgue convergence theorem; Vitali theorem(statement only); Functions of bounded variation; differentiation of an integral – equality of the derivative of an indefinite integral of an integrable function and the integrand a.e.

### **Course Outcomes:**

After going through this course a student must be able to

1. Explain upper & lower sums, Upper & lower integral & hence Riemann integral.
2. Develop the basic criterion for the existence of Riemann integral and connection between the existence of Riemann integral with monotonicity & continuity.



3. Differentiate between point wise & uniform convergence of sequences & series of functions.
4. Elaborate Cauchy criterion for uniform convergence of sequences & series of functions & hence connection of uniform convergence with differentiation integration & continuity.
5. explain the convergence and absolute convergence of improper integral of both type –I & II
6. explain the concepts of measurable sets, measurable functions with their basic properties.
7. Describe the integral of a a measureable function with their properties.
8. Explain the fundamental theorems such as Fatou’s lemma, monotone convergence theorem, vitalis theorem, Lebesgue convergence theorem etc.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

1. **Bilodeau, G. G., Thie, P. R. and Keough, G.E (2010),** An Introduction to Analysis, **second edition, Joes and Bartlett Learning.**
2. **Royden, H.L., (2006),** Real Analysis, **3<sup>rd</sup> edition, Prentice-hall of India Private Limited**

**TEXT BOOKS:**

**Reference Books:**

1. **Denlinger, C. G. (2011),** Elements of Real Analysis, **First Indian edition, Joes and Bartlett Learning.**
2. **Rudin, W., (1976),** Principles of Mathematical Analysis, 3<sup>rd</sup> edition, **McGraw Hill International Edition.**
3. **Yeh, J., (2000),** Lectures on Real Analysis, **World Scientific.**

## SEMESTER I

**Course Title: Applied Numerical Analysis**  
**Course Code: MS -104**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to train students in numerical analysis techniques and their applications.*

### Unit I

**Error analysis and solutions of non-linear equations:** Binary and machine numbers; computer accuracy; computer floating point numbers; errors and their propagation; order of approximation; method of fixed point iteration for solving non-linear equations; Bisection method of Bolzano; method of false position; initial approximation and convergence criteria.

### Unit II

**Solutions of non-linear equations continued:** Slope method for finding roots; Newton-Raphson theorem; Secant method; Aitken's process; Jacobian; Siedel and Newton's method for system of non-linear equations.

### Unit III

**Interpolation and polynomial approximation:** Taylor series and calculations of functions; Horner's method for evaluating a polynomial; interpolation, Lagrange's approximation, error terms and error bounds for Lagrange's interpolation; Newton polynomials; divided differences; Pade approximation.

### Unit IV

**Curve fitting:** Least square line; power fit method; data linearization; non-linear least squares method; least squares parabolas; Polynomial niggles; interpolation; piece wise cubic splines; existence and construction of cubic splines clamped; parabolic terminates and end point curvature adjusted spline; minimum property of cubic splines; Bernstein and their properties; Bezier curves.

### Unit V

**Numerical differentiation and integration:** Approximation of derivative; central differentiation formulas; error analysis and step size; Richardson extrapolation; differentiation of Lagrange's and Newton polynomials; Newton-Cotes quadrature formulae; composite Trapezoidal and Simpson's rules and their error analysis ; recursive trapezoidal and Simpson's rules; Boole rules; Romberg Integration; adaptive curvature; Gauss - Legendre integration.

### Course Outcomes:

After studying this course a student should be able to

1. Solve algebraic transcendental equation using an appropriate numerical method.
2. Approximate a function using an appropriate numerical method.
3. explain how to fit experimental data into different curves.

4. explain the concept of Spline, Bernstein's Polynomials and Bezier curve.
5. Perform an error analysis for a given numerical method.
6. explain central differentiation formulas, Richardson's extrapolation, differentiation of Lagrange's and Newton's polynomials.
7. Explain Newton's cotes quarantine formulae such as, Trapezoidal, Simpson's rules, Boole's rules, Romberg integration and their error analysis.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Curtis, F. G. and Patrick, O. W., (1999),** Applied Numerical Analysis, 6<sup>th</sup> edition, **Pearson Education.**
2. **John, H. M. and Kurtis, D. F., (2007),** Numerical Methods using Matlab, 4<sup>th</sup> edition, **Prentice Hall of India Pvt. Limited, New Delhi.**

**REFERENCE BOOKS:**

1. **Burden, R. L. and Faires, J. D.,(2009),** Numerical Analysis, 7<sup>th</sup> edition, **CENAGE Learning India (Pvt) Ltd.**
2. **Golub, G. and Loan, C. V., (1996),** Matrix Computations, 3<sup>rd</sup> edition, **John Hopkins University Press.**
3. **Jain, M.K., Iyenger, S.R.K. and Jain, R.K., (2007),** Numerical Methods for Scientific and Engineering Computation, 5<sup>th</sup> edition, **New Age International Publication, New Delhi.**

## SEMESTER I

**Course Title: Computer Fundamentals and C-Programming**  
**Course Code: MS -105**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to create awareness among students about computer applications and programming through C-language that will enable them to solve mathematical models.*

### Unit I

**Computer fundamentals:** Block diagram of computer; characteristics of a computer; generation of computers; I/O devices; memory and its types; number system & conversions; disk operating system (DOS); working with DOS commands (Internal and External).

### Unit II

**Introduction to windows:** Customize desktop; working with folders; add printer; add & removing programs; working with word pad; fundamentals of MS-word; creating and formatting MS-word documents; creating & customizing tables; mail merge and using math equations; overview of MS-Excel; working with cells; creating and formatting worksheets; working with formulae bar; creating charts.

### Unit III

**Programming languages:** Introduction; history of C language; structure of C program; variables, constants, keywords, operators and data types in C; decision making statements- (if, if else, else if ladder, nested if, switch-case, break, continue, goto).

### Unit I V

**Array and function:** Loops in C; arrays (one dimensional and multidimensional arrays); string array; introduction to function-element of user-defined function (declaration, function calling, function definition); functions call by value & call by reference; recursive function.

### Unit V

**Structure and pointer:** Definitions; declaration structure variable; accessing structure members; array of structures; introduction to pointers-accessing the address of variables; declaration pointer variables; initialization of pointer variable; pointer arithmetic.

**Course outcomes:** After completing this course a student

1. should be able to explain the concepts of input and output devices of computer and their working.
2. should know the uses of different types of worksheets like WordPad, MS- office and excel sheet.
3. should be able to design programs connecting decision structures, loops and functions.
4. should be able to explain the difference between call by value and call by address.
5. should be able to explain the dynamic behavior of memory by the use of pointers.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Balaguruswamy, E., (2004)**, Programming in ANSI C, 4<sup>th</sup> edition, **Tata McGraw Hill**.
2. **Saxena, S., (2007)**, MS- Office for Everyone, 1<sup>st</sup> edition, **Vikas Publications, New Delhi**.
3. **Sinha, P.K., (2007)**, Computer Fundamentals, 4<sup>th</sup> edition, **BPB Publications, New Delhi**.
4. **Taxali, R.K., (2007)**, PC Software for Windows, 1<sup>st</sup> edition, **TMH, New Delhi**.

**REFERENCE BOOKS:**

1. **Basandra, K., (2008)**, Computers Today, 1<sup>st</sup> edition, **Galgotia publication, New Delhi**.
2. **Schiltz, H.,(2004)**, C: The Complete Reference,4<sup>th</sup> edition, **Tata McGraw Hill**.

## SEMESTER I

**Course Title: Lab course on MS-104 & MS-105**  
**Course Code: MS -106**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 50**  
**Sessional Assessment: 50**  
**Duration of Exam: 3 hours**

Each student is required to maintain a practical record book .

The course carries 100 marks. Two practical tests, one Internal and one External, are to be conducted, each carrying 50 marks. The marks in practical test will be divided into 4 parts, program code, program execution , Viva-Voce and practical record book as per the choice of examiner. The student has to pass both internal and external practical test separately scoring a minimum of 20 marks for each test.

### **Course outcomes:**

After completing this course a student

1. Should be able to appreciate the use of computers in engineering industry.
2. Should have developed in him / her the basic understanding of computers, the concept of algorithms and algorithmic thinking.
3. Should have developed in him / her the ability to analyze a problem and develop an algorithm to solve it.
4. Should know the use of the C - programming language to implement various algorithms.

## SEMESTER II

Course Code	Course Title	No. of Credits	Distribution of Marks		
			SA	UE	Total
<b>Core Courses</b>					
MS- 201	Numerical Linear Algebra	04	40	60	100
MS -202	Functional Analysis with Applications	04	40	60	100
MS -203	Abstract Algebra with Applications	04	40	60	100
MS -204	Complex Analysis with Applications	04	40	60	100
<b>Choice based open elective courses(Students are required to opt any one of the following courses)</b>					
IT. 202	Soft skills in Information Technology	04	40	60	100
Comp. 203	Computer Applications and Operations	04	40	60	100
Bio. 204	Fundamentals of Biotechnology	04	40	60	100
Bot. 205	Mysteries of Green Plants	04	40	60	100
Bot. 206	Botany in Rural Development	04	40	60	100
Zol. 207	Nutrition, Health and Hygiene	04	40	60	100
Arab. 208	Fundamentals of Arabic Language	04	40	60	100
Eng. 209	Fundamentals of English	04	40	60	100
Edu. 210	Higher Education	04	40	60	100
Eco. 211	Principles of Banking	04	40	60	100
HT. 212	Basics of Tourism and Travel Agencies	04	40	60	100
HT. 213	Tourism Resources of J and K	04	40	60	100
Mgt. 214	Business communication and soft skills	04	40	60	100
Edu-215	Instructional technology	04	40	60	100
<b>Lab Course</b>					
MS -205	MatLab	04	50	50	100
<b>Total</b>		<b>24</b>	<b>250</b>	<b>350</b>	<b>600</b>

**SA: Sessional Assessment**

**UE: University Examination**

## SEMESTER II

**Course Title: Numerical Linear Algebra**  
**Course Code: MS-201**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives

*The main objective of this course is to introduce students to the fundamentals of linear algebra and numerical solutions of problems of linear algebra.*

### Unit I

**Matrices:** Review of fundamental concepts of vector space; Matrix of a linear transformation; matrix of sum and composition of linear transformations; change of basis matrix; similar matrices; determinant of a matrix and its basic properties; permutation and its signature; uniqueness of determinant map.

### Unit II

**Spectral theory:** Eigen values and eigen vectors of a matrix and a linear transformation; algebraic and geometrical multiplicities of an eigen value; diagonalizable linear mapping/matrix; Cayley-Hamilton theorem; minimum polynomial of a matrix and its properties; invariant subspaces of a vector space; primary decomposition theorem (statement only) and its special cases; necessary and sufficient conditions for simultaneous diagonalization of two matrices.

### Unit III

**Canonical and bilinear forms:** Nilpotent linear transformations; existence of triangular matrix; Jordan decomposition theorem (statement only); index of a nilpotent linear transformation and its elementary properties; Jordan Block matrix; Jordan form; Jordan basis; bilinear form; symmetric and skew symmetric bilinear forms; quadratic form and its properties; Sylvester's theorem; positive definite quadratic form.

### Unit IV

**Numerical methods for linear systems:** Gauss elimination method; Gauss Jordan elimination method; pivoting; LU factorization method; Doolittle method; Crout's method; Cholisky's method; Jacobi iteration method; Gauss Seidel iteration method; matrix norms; introduction to ill conditioning; well conditioning and condition number of a matrix.

### Unit V

**Numerical methods for finding eigen values and eigenvectors:** Power method; shifted inverse power method; Jacobi's method; Householder's method; Householder's reflection theorem; Householder's transformation and its computation; QR method; Gerschgorian's theorem; Peron's theorem; Schur's theorem.

### Course outcomes

On successful completion of this course we expect a student will be able to

1. explain vector space, linear dependence / independence, basis and dimension, linear transformation, change of basis matrix, permutation and its signature.
2. explain the concept of characteristic polynomial to compute the eigen values and eigen vectors of a square matrix and Cayley-Hamilton theorem.
3. explain the concept of minimum polynomial of a matrix and its properties, primary decomposition theorem and diagonalization.



4. explain the concept of Nilpotent linear transformations, Jordan decomposition theorem, Jordan Block Matrix, Jordan form, Jordan basis.
5. explain the concept of bilinear forms, symmetric and skew symmetric bilinear forms, quadratic form and its properties.
6. explain the numerical methods such as Gauss- Jordan elimination method, LU factorization method, Doolittle method, Crout's method, Cholsky's method, Gauss-Seided iteration method for solving the system of linear equations.
7. explain the numerical methods such as power method, Jocabi's method, Household's method, QR method and theorems such as Gerschgorian's theorem, person's theorem.

**Note for Paper Setting:**

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**Books Recommended:**

**TEXT BOOKS:**

1. **Blyth, T.S. and Robertson, E. F., (2007),** Basic Linear Algebra, 2<sup>nd</sup> Edition, **Spinger.**
2. **Blyth, T.S. and Robertson, E. F., (2008),** Further Linear Algebra, 2<sup>nd</sup> Edition, **Spinger.**
3. **John, H. M. and Kurtis, D. F., (2007),** Numerical Methods using Matlab, 4th edition, **Prentice Hall of India Pvt. Limited, New Delhi.**

**REFERENCE BOOKS:**

1. **Burden, R. L. and Faires, J. D., (2009),** Numerical Analysis, 7th edition, **CENAGE Learning India (Pvt) Ltd.**
2. **Golub, G. and Loan, C. Van, (1996),**Matrix Computations, 3<sup>rd</sup> edition, **John Hopkins University Press.**
3. **Jain, M.K., Iyenger, S.R.K. and Jain, R.K., (2007),** Numerical Methods for Scientific and Engineering Computation, 5<sup>th</sup> edition, **New Age International Publication, New Delhi.**
4. **Kreyszig, E.,** Advanced Engineering Mathematics, 8<sup>th</sup> Edition, **Wiley India Private limited.**

## SEMESTER II

**Course Title: Functional Analysis with Applications**

**Course Code: MS -202**

**Credits: 4**

**Maximum Marks: 100**

**University Examination: 60**

**Sessional Assessment: 40**

**Duration of Exam: 3 hours**

### Objectives

*The main objective of this course is to introduce students to the fundamentals of functional analysis and make them aware of its applications.*

### UNIT I

**Normed spaces:** Definition, examples and basic properties of normed spaces; completeness and equivalence of norms on finite dimensional normed spaces; characterization of compact sets in finite dimensional normed spaces; Riesz lemma; introduction to  $\mathbb{L}^p$  -spaces.

### UNIT II

**Linear operators on normed spaces:** Definition and basic properties of bounded linear operators; connection between continuity and boundedness of linear operators; continuity of linear operators on finite dimensional spaces; completeness of normed space of operators; dual spaces of  $\mathbb{R}^n$  and  $l^p$  spaces, Hahn Banach extension theorem for normed spaces and its consequences.

### UNIT III

**Inner product spaces(IPS):** Definition and basic properties of IPS; Hilbert spaces; existence of minimizing vector; orthogonality; Projection theorem; orthogonal complement of a set and its basic properties; Bessel's inequality; total orthonormal sets; Parseval's relation; connection between separability and orthonormal sets; isomorphism of Hilbert spaces of same dimension.

### UNIT IV

**Inner product spaces and Banach fixed point Theorem:** Riesz theorem; sesquilinear forms; Riesz representation for sesquilinear forms; Hilbert adjoint operator and its basic properties; basic properties of self adjoint, unitary and normal operators; Banach fixed point theorem and its applications to differential and integral equations –Picard's existence and uniqueness theorem, Fredholm and Volterra integral equations.

### UNIT V

**Reflexive spaces and fundamental theorems:** Reflexive spaces; Hilbert spaces and finite dimensional normed spaces as examples of reflexive spaces; separability of dual normed space as a sufficient condition for the separability of the normed space; Baire's Category theorem; uniform boundedness theorem and its application to space of polynomials and Fourier Series; Open mapping and closed graph theorems.

### **Course Outcomes:**

On successful completion of this course, we expect a student

1. should be able to explain the concept of inner product and norm on a vector space.

2. should be able to explain the concept of normed, Banach & Hilbert spaces with standard examples and relation between them.
3. should be able to explain the concepts of bounded linear operator & bounded linear functional with standard examples.
4. should be able to explain the properties of linear operators on finite and infinite dimensional normed spaces.
5. should be able to explain the dual spaces of  $\mathbb{R}^n$  and  $l^p$  spaces and completeness of the normed space of operators.
6. should know the Banach contraction principle with applications to differential & integral equations.
7. should know the fundamental theorems such as Riez Lemma, Hahn Banach extension theorem, closed graph theorem, open mapping theorem, Principle of uniform boundedness, Bessel's inequality, projection theorem, Parseval's relation, Baire Category theorem and Riesz theorem with applications.
8. should be able to explain the concept of separable and reflexive normed spaces.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Kreyszig, E., (2006)**, Introductory Functional Analysis with Applications, 1<sup>st</sup> edition, **Wiley Student edition.**

**REFERENCE BOOKS:**

1. **Bachman, G. and Narici, L., (1966)**, Functional Analysis, **Academic Press New York.**
2. **Cheney, W., (2000)**, Analysis for Applied Mathematics, **Springer.**
3. **Rynne, B. P. and Youngson, M. A., (2008)**, Linear Functional Analysis, 2<sup>nd</sup> edition, **Springer.**
4. **Siddiqi, A. H., (2004)**, Applied Functional Analysis, **Marcel-Dekker, New York.**

## SEMESTER II

**Course Title: Abstract Algebra with Applications**

**Course Code: MS -203**

**Credits: 4**

**Maximum Marks: 100**

**University Examination: 60**

**Internal Assessments: 40**

**Duration of Exam: 3 hours**

### Objectives

*The main objective of this course is to introduce students to the fundamentals of abstract algebra- group and ring theory with their applications to coding theory.*

### Unit I

**Class equation and Sylow's theorem with applications:** Conjugate of an element of a group; class equation and its applications – non-triviality of centre of a group of order  $p^n$ , Cauchy's theorem; number of a conjugate classes in  $S_n$ ; 1<sup>st</sup> part of Sylow's theorem (Proof by induction); 2<sup>nd</sup> and 3<sup>rd</sup> parts of Sylow's theorem (Proofs not included); Applications of Sylow's theorem in the determination of simplicity of groups of order 72, 20449, 225, 30, 385, 108,  $p^2q$  ( $p, q$  primes) and 60.

### Unit II

**Ring theory:** Definition and examples of rings; special classes of rings – integral domain, field; characteristic of an integral domain; Homomorphism; ideals and quotient rings; maximal ideals; the field of quotient of an integral domain.

### Unit III

**Euclidean rings:** Euclidean ring (ER); ideals in a ER; principle ideal ring; concept of division, gcd, units, associate and prime elements in a ER; relation between prime elements and maximal ideals in a ER; ring of Gaussian integers and ring of polynomials  $F[x]$ ,  $F$  a field as examples of ERs.

### Unit IV

**Polynomial Rings and UFD:** Polynomials over the rational field; primitive polynomials; content of a polynomial; Gauss lemma; Einstein's criteria; polynomial rings over commutative rings; UFD and its relation with ER;  $R[x]$  as a UFD when  $R$  is a UFD; relation between PIR and UFD.

### Unit V

**Algebraic coding theory: Classification, structure and subfields of a finite field; Linear codes; Hamming distance and weight with properties; correcting capability of a linear code; orthogonality relation; Parity check matrix decoding; coset decoding; syndrome.**

### Course Outcomes

After completing this course, we expect a student have understood

1. Class equation with applications, Cauchy theorem, Sylow's theorems with applications to find simplicity of a group.
2. The concept of ring with standard examples, different classes of rings such as Integral domain, field, ideal and quotient ring.
3. The concept of ideal with standard examples, maximal and prime ideals and quotient field of an Integral domain.
4. The concept of Unique factorization domain, Euclidean ring and Principal Integral domain and relation between them.
5. The concept of Ring of Gaussian integers and polynomials with properties.
6. Gauss lemma and Eisenstein's criteria.
7. the characterization of subfields of a finite field.
8. The concept of linear code, Hamming distance, coding, decoding, and syndrome.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Gallian, J. A. (1998)**, Contemporary Abstract algebra, **Fourth edition**, Narosa.
2. **Herstein, I.N.(2004)**, Topics in Algebra, 2<sup>nd</sup>edition, **Wiley**.

**REFERENCE BOOKS:**

1. **Artin, M., (2010)**, Algebra, 2<sup>nd</sup> edition, **Springer**.
2. **Farmer, D.W., (1963)**, Groups and Symmetry: A Guide to Discovering Mathematics, **American Mathematical Society**.
3. **Jacobs, H. R., (1979)**, Elementary Algebra, 1<sup>st</sup>edition, **W. H. Freeman**.
4. **Levinson, N., (1970)**, Coding Theory: A Counter Example to G. H. Hardy's Conception of Applied Mathematics, AMS Monthly 77: 249-258

## SEMESTER II

**Course Title: Complex Analysis with Applications**

**Course Code: MS -204**

**Credits: 4**

**Maximum Marks: 100**

**University Examination: 60**

**Sessional Assessment: 40**

**Duration of Exam: 3 hours**

### Objectives

*The objective of this course is to introduce students to the fundamentals of Complex analysis (with applications) which is a tool with remarkable and almost mysterious utility in applied mathematics.*

### Unit I

**Analytic functions:** Extended complex plane; derivative of a complex function and its basic properties; C-R equations in both cartesian and polar form; analytic and harmonic functions; exponential, trigonometric and hyperbolic functions; branch of a multi-valued function; logarithmic function; complex exponents.

### Unit II

**Integral of a complex function-I:** Contour integral and its basic properties; ML-inequality; primitives; Cauchy-Goursat theorem for triangles, open convex sets and simply connected domains; winding number; Cauchy's integral formula; derivative of an analytic function; Morera's theorem; Cauchy's inequality; Liouville's theorem; fundamental theorem of Algebra.

### Unit III

**Integral of a complex function-II:** Convex Hull; Gauss theorem; Luca's theorem; Gauss mean value property; max/min modulus principle; Taylor's series; Parseval's formula; zeros of an analytic function; Schwartz lemma; Borel Caratheodary theorem; reflection principle; Laurant's series.

### Unit IV

**Singularities and residues:** Isolated singularities; Riemann theorem; residues; residue theorem; connection between zeroes and poles; Casorati-Weirstrass theorem; meromorphic functions; argument principle; Rouche's theorem; Hurwitz theorem; definite integral involving sines and cosines; improper integrals involving rational functions; improper integrals involving sines and cosines; Jordan's inequality and Jordan's lemma.

### Unit V

**Conformal mapping:** Linear and reciprocal transformations; square map, conformal and isogonal maps; conformality theorem; Bi-linear transformation and its basic properties; fixed points of a bilinear transformation; cross ratio; exponential and trigonometric transformations; Riemann mapping theorem (proof not included); construction of harmonic functions; Poission integral formula.

### Course Outcomes

After the completion of this course a student must be able to

1. Explain the concept of extended complex plane, derivative of a complex function with its basic properties, analytic function, Cauchy Riemann equations.

2. Explain in detail the elementary complex functions such as exponential, trigonometric, hyperbolic, logarithmic, etc.
3. Describe contour integral, convex hull, open convex sets, simple connected domains & winding number etc.
4. Provide the proof of theorems like Cauchy-Goursat theorem, Cauchy integral formula, Cauchy inequality, Morera's theorem, Liouville's theorem, fundamental theorem of Algebra, maximum, minimum modulus theorem, Schwarz lemma, Borel Carathéodory theorem, reflection principle etc.
5. Differentiate between isolated and non-isolated singularities, zeroes and poles and should be able to find residues.
6. Explain the theorems like Riemann theorem, Residue theorem, Cauchy's theorem, Weierstrass theorem, argument principle, Hurwitz theorem, Jordan's lemma, Poisson integral formula, Riemann mapping theorem etc.
7. Find real integrals by using complex analysis techniques and construction of harmonic functions.
8. Describe Bi-linear transformation with its basic properties and the concept of cross ratios.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books recommended:**

**TEXT BOOKS:**

1. **Kasana, H. S., (2012),** Complex Variables, Theory and Applications, 2<sup>nd</sup> Edition, **PHI learning Private limited, New Delhi-110001.**
2. **Mathews, J. H. & Howell, R. W., (2006),** Complex Analysis for Mathematics and Engineering, 5<sup>th</sup> edition, **B Jones and Bartlett Publishers**

**REFERENCE BOOKS:**

1. **Brown, J. W. and Churchill, R. V., (2009),** Complex Variables and Applications, 8<sup>th</sup> Edition, **McGraw-Hill International.**
2. **Conway, J. B., (1973),** Functions of one Complex Variable, 2<sup>nd</sup> edition, **Springer International Student edition.**
3. **Rudin, W., (1987),** Real and Complex Analysis, 3<sup>rd</sup> edition, **McGraw Hill International Edition.**

## SEMESTER II

**Course Title: MatLab**  
**Course Code: MS -205**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 50**  
**Sessional Assessment: 50**  
**Duration of Exam: 3 hours**

### **Objectives**

The Lab course has been designed to train students of Mathematics in using MatLab and computers in evolving solutions to problems of Numerical Analysis and linear algebra.

Each student is required to maintain a practical record book .

The course carries 100 marks. Two practical tests, one Internal and one External, are to be conducted, each carrying 50 marks. The marks in practical test will be divided into 4 parts, program code, program execution , Viva-Voce and practical record book as per the choice of examiner. The student has to pass both internal and external practical test separately scoring a minimum of 20 marks for each test.

### **Course Outcomes**

After studying this course, we expect a student have understood

1. the applicability of MATLAB in Mathematics in particular and engineering applications in general.
2. the commands of MATLAB which one uses to solve elementary problems of numerical Analysis.
3. the concept of M-file and Script file along with control flow programming.
4. the plotting of graphs of functions by using syntax and semantics.



### SEMESTER III

Course Code	Course Title	No. of Credits	Distribution of Marks		
			SA	UE	Total
<b>Core courses</b>					
MS-301	Advanced Topics in Topology	04	40	60	100
MS-302	Theory of Operators	04	40	60	100
MS-303	Calculus in $\mathbb{R}^n$	04	40	60	100
MS-304	Set Theory	02	20	30	50
MS-305	Lab course on LATEX	02	25	25	50
<b>Choice based Complementary Electives</b> Students are required to choose any two of the following courses					
MS-306	Differential Geometry	04	40	60	100
MS-307	Number Theory	04	40	60	100
MS-308	Module Theory	04	40	60	100
MS-309	Commutative Algebra	04	40	60	100
MS-310	Advanced Complex Analysis	04	40	60	100
MS-311	Abstract Measure Theory and Integration	04	40	60	100
<b>Total</b>		<b>24</b>	<b>245</b>	<b>355</b>	<b>600</b>

**SA: Sessional Assessment**  
**UE: University Examination**

## SEMESTER III

**Course Title: Advanced Topics in Topology**  
**Course Code: MS-301**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to familiarize the students with the advanced topics of topology.*

### Unit I

**Compact spaces:** Compact spaces; finite intersection property; Limit point compact and sequentially compact spaces and their connection with compactness; locally compact spaces; one point compactification; first and 2<sup>nd</sup> countable spaces; Lindeloff spaces; regular and normal spaces and products of regular, normal and Hausdorff spaces.

### Unit II

**Normal spaces:** Connection of regular; metrizable; compact Hausdorff spaces and well ordered sets with normal spaces; Urysohn's Lemma; subspaces and product of completely regular spaces; Urysohn's metrization theorem; imbedding theorem; Tietz extension theorem; Tychonoff theorem

### Unit III

**Nets, filters and Manifolds:** Nets, subnets, cluster point of a net, convergence of a net and continuous maps, nets in product spaces, filters and their convergence, filter base, filter in product spaces ultra filters, relationship between nets and filters; imbedding of manifolds –  $m$  – manifolds; finite partitions of unity and its existence; embedding of a compact  $m$  – manifold in to  $\mathbb{R}^n$

### Unit IV

**Stone– Cech Compactification and metrization theorem:** Compactification of a space and its relation with continuous maps; stone-Cech compactification; open covering of a metrizable space;  $G_\delta$  –sets; relation between regular and normal space via countably locally finite basis; Nagata- Smirnov metrization theorem

### Unit V

**Para-compactness, Smirnov metrization theorem and compact spaces:** Para-compact space and its relation with normed space; closed subspaces of para-compactness; Lemma of E. Michael; Relation of para-compact space with metrizable and regular Lindelof spaces; Smirnov metrization theorem; totally bounded sets in a metric space; characterization of compactness in terms of complete and totally bounded sets; equicontinuous family; classical version of Ascoli's theorem(statement only)

### Course Outcomes:

After completing this course a student should be able to

1. explain the concepts of Compactness, Limit point compactness, local compactness, sequential compactness and relations between them.

2. explain the concepts of first and 2<sup>nd</sup> countable spaces, Lindeloff spaces regular spaces, normal spaces, metrizable spaces, compact Hausdorff spaces, Para-compact spaces normed spaces and relations between them.
3. explain the fundamental theorems such as Urysohn's Lemma, Urysohn's metrization theorem, imbedding theorem, Tietz extension theorem, Tychonoff theorem, Nagata- Smirnov metrization theorem, Lemma of E. Michael, Smirnov metrization theorem, Ascoli's theorem.
4. explain the concepts and examples of Nets, subnets, filters, subfilters and connection between them.
5. explain the concepts of convergence of a net and filter and their relationship with continuity.
6. explain the concepts of manifolds and embedding of a compact manifold in  $\mathbb{R}^n$ .
7. explain the concept of Compactification of a space and its relation with continuous maps.
8. Explain the concepts of totally boundedness and completeness in metric spaces and their relationship with compactness.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books recommended:**

**TEXT BOOKS:**

1. **Munkers J.R. ,(2000),** Topology, 2<sup>nd</sup> Edition, **PHI.**

**REFERENCE BOOKS:**

1. **Kelley, J. L., (1975),** General Topology, **Springer.**
2. **Willard S., (2010),** General Topology, **Dover Publications New York.**

## SEMESTER III

**Course Title: Theory of Operators**  
**Course Code: MS-302**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to study spectral properties of operators on normed spaces.*

### Unit I

**Spectral theory of Linear operators in normed spaces:** Spectrum and resolvent of a bounded linear operator; non-emptiness, closedness and boundedness of the spectrum of bounded linear operator; spectral mapping theorem for polynomials; spectral radius

### Unit II

**Compact linear operators(CLO) on normed spaces and their spectrum-I:** CLO and its connection with continuity, dimension and weak convergence; compactness of limit of a sequence of CLO; separability of range; compactness of extension and adjoint; countability of spectrum; compactness of product of two CLO ; null spaces and range of  $T-\lambda I$ ; relation between spectral value and eigen value

### Unit III

**Compact linear operators on a normed spaces and their spectrum-II:** Adjoint of an operator on a normed space and its basic properties; operator equations - existence of solution, bounds on solutions; theorems of Fredholm type; Fredholm alternative – Fredholm alternative for integral equations, compact integral operator

### Unit IV

**Spectral theory of bounded self adjoint linear operators -I:** Basic properties of eigen values and eigen vectors; resolvent set; realness of spectrum; spectrum bounds and their relation with norm of the operator; emptiness of residual spectrum; positive operator and their product; monotone sequence of operators; square root of a positive operator; projection operators; sum and product of projections

### Unit V

**Spectral theory of bounded self-adjoint linear operators(BSALO) –II:** Difference of projections; monotone sequence of projections; spectral family of (BSALO); projection of +ve and -ve parts of operators; spectral family associated with an operator; spectral theorem for (BSALO); properties of polynomial of a (BSALO); extension properties of the spectral family of (BSALO)

### **Course Outcomes:**

*On successful completion of this course, we expect a student*

- 1. should be able to explain the concept of spectrum of a bounded linear operator(BLO) with examples and properties such as compactness and spectral radius.*
- 2. should be able to explain the spectral mapping theorem for polynomials, concept of compact linear operator(CLO), its basic properties and its connection with BLOs and weak convergence.*

3. should be able to explain the compactness of adjoint of CLO and compactness of product of two CLOs.
4. should be able to explain the cardinality of spectrum and relation between spectral values and eigen values of a CLO.
5. should be able to explain the basic spectral properties of a self adjoint BLOs such as realness of the spectrum, spectrum bounds and their relationship with norm of the operator and emptiness of residual spectrum.
6. Should be able to explain the concept and properties of positive operator, square root of a positive operator, projection operators and their properties such as sum, difference and product.
7. should be able to explain the concept and properties of spectral family of a self adjoint BLOs with properties.
8. should be able to explain the concepts of +ve and -ve parts of an operator and their basic properties.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Kreyszig, E., (2005)**, Introductory Functional Analysis with Applications, 1<sup>st</sup> edition, **Wiley Student edition.**

**REFERENCE BOOKS:**

1. **Conway, J. B., (2000)**, A Course in Operator Theory, 2<sup>nd</sup> edition, **American Mathematical Society.**
2. **Douglas, R. G., (2008)**, Banach Algebra Techniques in Operator Theory, 2<sup>nd</sup> edition, **Springer.**

## SEMESTER III

**Course Title: Calculus in  $\mathbb{R}^n$**   
**Course Code: MS-303**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives

*The aim of this course is to introduce the students differential and integral calculus in  $\mathbb{R}^n$  with an introduction to distribution theory.*

### Unit I

**Differential Calculus-I:** Directional derivatives, Partial Derivatives, Total Derivatives and their connection with continuity, The Jacobian matrix; Chain rule, mean value theorem; connection between total & Partial derivatives; Equality of Partial derivatives.

### Unit II

**Differential Calculus-II:** Taylor's formula for real valued functions of several variables; Properties of functions with non-zero Jacobian determinant; Inverse function and implicit function theorems; Extremum of real valued functions for several variables- 2<sup>nd</sup> derivative test; Lagrange's multipliers.

### Unit III

**Integral calculus -I:** Iterated integrals; multiple Riemann integral; equality of iterated and multiple integral; basic properties of Riemann integral; Leibnitz rule; change of variable

### Unit IV

**Integral calculus and test functions:** Definition and examples of improper Riemann integral; independence of the value of improper integral over a sequence of sets; comparison tests for the existence of improper integral; Definition and examples of test functions; convergence in the space  $\mathcal{D}(\mathbb{R}^n)$  of test functions

### Unit V

**Distributions:** Definition and examples of distributions---regular; Dirac delta; Heaviside distribution; derivative of a distribution; convergence of distributions; product of a  $C^\infty(\mathbb{R}^n)$  function and a distribution; convolution of a test function and a distribution

### Course outcomes

After studying this course we expect student should be able to explain

1. the concept of continuity, directional, partial and total derivatives and their relationships with each other.
2. the fundamental theorems such as Chain rule, mean value theorem, Taylor's theorem, Inverse and Implicit function theorems and their applications.

3. The concept of the Jacobian matrix, the condition of equality of mixed partial derivatives, the concept of Extreme-Values of multi-variable functions and Lagrange's multipliers.
4. The concept and properties of multiple integrals, iterated integrals and relationship between them.
5. The concept of improper integrals and various convergence tests such as the comparison test.
6. The concept and examples of test functions, distributions such as regular, Dirac delta, Heaviside.
7. The concepts of derivative of a distribution and convergence of distributions.
8. The product of a  $C^\infty(\mathbb{R}^n)$  function and a distribution and convolution of a test function with a distribution.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

3. **Apostol, Tom M., (2002),** Mathematical Analysis, 1<sup>st</sup> edition, **Narosa Publishing House.**
4. **Cheney, W., (2000),** Analysis for Applied Mathematics, **Springer, New York.**
5. **Richard, E. W., Richard, H. R. and Hale, F. T., (1972),** Calculus of Vector Functions, 3<sup>rd</sup> edition, **Prentice Hall.**

**REFERENCE BOOKS:**

4. **Ghorpade, S.R and Limaye, V.B.,(2010),** A course in Multivariable calculus and Analysis, **Springer.**
5. **Rudin, W., (1976),** Principles of Mathematical Analysis, 3<sup>rd</sup> edition, **McGraw Hill International Edition.**

## SEMESTER III

**Course Title: Set Theory**  
**Course Code: MS-304**  
**Credits: 2**

**Maximum Marks: 50**  
**University Examination: 30**  
**Sessional Assessment: 20**  
**Duration of Exam: 3 hours**

### **Objectives:**

*The aim of this course is to introduce the students with the ideas of advanced set theory.*

### **Unit I**

**Countability of sets:** Sets, relations, functions and their basic properties; definitions, examples and properties of finite, countable and uncountable sets

### **Unit II**

**Cardinal and ordinal numbers:** Cardinal arithmetic; the cardinality of the continuum; Well ordered sets; ordinal numbers; axiom of replacement; transfinite induction and recursion; ordinal arithmetic.

### **Unit III**

**Alephs and axiom of choice:** Initial ordinals; addition and multiplication of alephs; the axiom of choice and its equivalent forms; infinite sums and products of cardinal numbers; regular and singular cardinals; exponentiation of cardinals.

### **Course Outcomes**

After Studying this course a student is expected to

1. explain the basic difference between finite, infinite, countable, uncountable sets and their various properties.
2. explain the arithmetic of cardinal and ordinal numbers.
3. explain the concept and examples of well ordered sets.
4. explain axiom of replacement and transfinite induction and recursion.
5. explain the axiom of choice and its various equivalent forms.

### **Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 06 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 06 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 08 marks.*

### **Books Recommended:**

### **TEXT BOOKS:**



## **REFERENCE BOOKS:**

1. **Hrbalek, K. and Jech, T.,(1999),** Introduction to set theory,3<sup>rd</sup> edition, **Marcel Dekker, Ind.**
- 2 **Halmos, P., (2011),** Naïve set theory, **Martino Fine Books.**

## SEMESTER III

**Course Title: *Lab course on LATEX***  
**Course Code: *MAM-305***  
**Credits: 2**

**Maximum Marks: 50**  
**University Examination: 25**  
**Sessional Assessment: 25**  
**Duration of Exam: 3 hours**

### **Objectives:**

*The objectives of this course is to train he students in LATEX.*

Each student is required to maintain a practical record book .

The course carries 50 marks. Two practical tests, one Internal and one External, are to be conducted, each carrying 25 marks. The marks in practical test will be divided into 4 parts, program code, program execution , Viva-Voce and practical record book as per the choice of examiner. The student has to pass both internal and external practical test separately scoring a minimum of 10 marks for each test.

### **Course Outcomes**

After studying this course, we expect a student have understood

1. Typeset mathematical formulae using latex.
2. Use the preamble of Latex file to define document class and layout options.
3. Use nested list and enumerate environment within a document.
4. Use tabular and array environment within latex document.
5. Use various methods to either create or import graphics into a Latex document.

## SEMESTER III

**Course Title: Differential Geometry**  
**Course Code: MS-306**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study geometry in Euclidean space with the help of calculus.*

### Unit I

**Differential calculus in  $\mathbb{R}^n$ :** Differential calculus in  $\mathbb{R}^n$ ; diffeomorphism; tangent space of  $\mathbb{R}^n$ ; vector fields on  $\mathbb{R}^n$ ; natural frame field; dual vector space; gradient vector field; directional derivative; curve of class  $C^k$ .

### Unit II

**Differential forms and manifolds:** Integral curve; local flow; derivative map; covariant derivative; cotangent space and differential forms on  $R^n$ ; Lie bracket; charts and atlases; differential manifolds.

### Unit III

**Topology on manifolds:** Induced topology on manifolds; functions and maps; some special functions of class  $C^\infty$ ; para-compact manifolds; pullback functions; tangent vectors and tangent space; tangent bundle; pullback vector fields.

### Unit IV

**Tensors-I:** Multi-linear functions and tensors; tensor product; tensor fields; tensors on finite dimensional vector spaces; tensors of type  $(p,q)$ ; connections; torsion tensor; curvature tensor.

### Unit V

**Tensors-II:** Contraction; Concepts of symmetric and alternating tensors and basic properties; Bianchi and Ricci identities; concept of geodesics; concept of Riemannian manifold.

### Course Outcomes:

After completing this course a student should be able to

1. explain the concepts of diffeomorphism, tangent space and vector fields on  $\mathbb{R}^n$ , natural frame field, gradient vector field, and curves of class  $C^k$ .
2. explain the concepts of integral curve, local flow, derivative map, cotangent space and differential forms on  $R^n$ , Lie bracket, charts atlases.
3. explain the concepts of differential manifolds, induced topology on manifolds and para-compact manifolds.
4. explain the concepts of pullback functions, tangent vectors and tangent space, tangent bundle and pullback vector fields.
5. explain the concept of tensor, tensor product, tensor field, torsion tensor; curvature tensor and tensors of type  $(p, q)$ .
6. explain the properties of tensors on finite dimensional vector spaces.

7. explain the concept of symmetric and alternating tensors and their basic properties
8. explain the Bianchi and Ricci identities and the concept of geodesics and Riemannian manifold.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Amur, K. S., Shetty, D. J. and Bagewadi, C. S.,(2010),**An Introduction to Differential Geometry, **Narosa Publishing house.**

**REFERENCE BOOKS:**

1. **De, U. C. and Shaikh,A. A.,(2009),** Differential Geometry of Manifolds, **Narosa Pub. House.**
2. **Neill, B. O., (1966),**Elementary Differential Geometry, **Academic Press, New York.**
3. **Thorpe, J. A., (1979),**Elementary Topics in Differential Geometry, Undergraduate Text in Mathematics, **Springer Verlag.**
4. **Somasundaram, D., (2010),** Differential Geometry: A First Course, **Narosa Pub. House.**

## SEMESTER III

**Course Title: Number Theory**  
**Course Code: MS-307**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to familiarize the students with numbers and their properties.*

### Unit I

**Divisibility:** Euclidean algorithm; primes; congruences; Fermat's theorem; Euler's theorem ; Wilson's theorem; Fermat's quotients and their elementary consequences; solutions of congruences; Chinese remainder theorem; Euler's phi-function.

### Unit II

**Congruence:** Congruence modulo powers of prime; power residues; primitive roots and their existence; quadratic residues; Legendre's symbol; Gauss lemma about Legendre's symbol; quadratic reciprocity law; proofs of various formulations; Jacobi symbol.

### Unit III

**Arithmetic Functions:** Greatest integer function; arithmetic functions; multiplicative arithmetic functions(elementary ones); Mobius inversion formula; convolution of arithmetic functions; group properties of arithmetic functions; recurrence functions; fibonacci numbers and their elementary properties.

### Unit IV

**Diophantine equations:** Diophantine equations – solutions of  $ax + by = c$ ,  $x^2 + y^2 = z^2$ ,  $x^4 + y^4 = z^2$ ; properties of Pythagorean triplets; sums of two, four and five squares; assorted examples of diophantine equations.

### Unit V

**Continued fractions:** Simple continued fractions; finite and infinite continued fractions; uniqueness; representation of rational and irrational numbers as simple continued fractions; rational approximation to irrational numbers; Hurwitz theorem; basic facts of periodic continued fractions and their illustrations (without proofs); Pell's equation.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain Euclidean algorithm, Euler's Phi function and some fundamental theorems such as Fermat's theorem, Euler's theorem, Wilson's theorem Chinese remainder theorem, Gauss lemma, quadratic reciprocity law.
2. explain the concepts of power residues, Primitive roots, Legendre's symbols and Jacobi symbols.
3. explain the concept and properties of arithmetic functions and Fibonacci numbers.

4. explain Mobius inversion formulae, Diophantine equations, Pythagorean triplets and Fermat's last theorem.
5. explain the simple continued fractions, finite and infinite continued fractions, rational and irrational numbers as simple continued fractions.
6. Explain the Hurwitz theorem, periodic continued fractions and Pell's equation.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Niven, I., Zuckerman, H. S. and Montgomery, H. L., (2003),** An Introduction to the Theory of Numbers, 6th edition, **John Wiley and sons, Inc., New York.**
2. **Burton, D. M., (2002),** Elementary Number Theory, 4th edition, **Universal Book Stall, New Delhi.**

**REFERENCE BOOKS:**

1. **Dickson, L. E., (1971),** History of the Theory of Numbers , Vol. II, Diophantine Analysis, **Chelsea Publishing Company, New York.**
2. **Hardy, G. H. and Wright, E. M., (1998),** An Introduction to the Theory of Numbers, 6th edition, **The English Language Society and Oxford University Press.**
3. **Niven, I., Zuckerman, H. S. , (1993),** An Introduction to the Theory of Numbers, 3rd edition, **Wiley Eastern Ltd., New Delhi.**

## SEMESTER III

**Course Title: Module Theory**  
**Course Code: MS-308**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study modules and their properties.*

### Unit I

**Fundamentals of modules:** Left modules and right modules; examples of modules; submodules; intersection, union and sum of sub-modules of a module; finitely generated module; homomorphism; fundamental theorems on homomorphism and quotient modules.

### Unit II

**Free modules-I:** Direct sum of modules; equivalent condition for direct sum; free modules; characterization of free modules; cardinality basis of a module; rank of finitely generated free module; simple and semisimple modules.

### Unit III

**Free modules-II:** Finitely generated free module over PID; the invariant factor decomposition; structure theorem for finitely generated modulus over a PID; torsion module and torsion free module; condition for a finitely generated module over a PID to be free module; the primary decomposition theorem; Chinese remainder theorem.

### Unit IV

**Projective and injective modules:** Exact sequences; projective modules; characterization of projective modules; condition for a ring to be semi-simple ring; injective modules; characterization of injective modules; Baer's criterion; injective hull; Noetherian rings; necessary and sufficient condition for a ring to be Noetherian ring;

### Unit V

**Simple rings:** Simple ring; Schin's lemma; semi-simple modules; the Astin-Wedder Burn theorem; simple modules; Jacobson radical; Astinan ring; Hopkins Levitzki theorem.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain the concept, examples and basic properties of modules, submodules, quotient modules, simple and semi - simple modules.
2. explain the fundamental theorems on homomorphism between modules.
3. explain the concept of free modules (its various characterizations) and rank of finitely generated free modules.
4. explain the concepts of Finitely generated free module over PID, torsion module and torsion free module and the invariant factor decomposition.

5. explain some fundamental results such as structure theorem for finitely generated module over a PID, condition for a finitely generated module over a PID to be free module, the primary decomposition theorem and Chinese remainder theorem.
6. explain the concepts, examples and properties of projective and injective modules.
7. Explain the concept, examples and properties of Simple ring, Noetherian rings and semi-simple modules.
8. Explain some fundamental results such as condition for a ring to be semi-simple ring, necessary and sufficient condition for a ring to be Noetherian ring, Baer's criterion, Schin's lemma, Artin-Wedder Burn theorem, Hopkins Levitzki theorem.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Grillet, P. A., (2007)**, Abstract Algebra: Graduate Texts in Mathematics, 2<sup>nd</sup> edition, Springer.

**REFERENCE BOOKS:**

1. **Blyth, T.S., (1982)**, Module Theory: An Approach to Linear Algebra, **Oxford University Press**.
2. **Albu, T., Birkenmeier, G.F., Erdogan, A. and Tercan, A., (2010)**, Rings and Module Theory, **Birkhäuser Basel**.



## SEMESTER III

**Course Title: Commutative Algebra**  
**Course Code: MM-309**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study ideals, modules and rings.*

### Unit I

**Ideals:** Ring, ring homomorphism, ideals, operation on ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local ring, Nilradical and Jacobson radical, exercises based on above topics.

### Unit II

**Modules:** Module homomorphism, Submodules, Quotient modules, Operation on submodules, direct sum and product of modules, Finitely generated modules; Nakayama lemma, Tensor product of modules, Exercises based on the above topics.

### Unit III

**Localization and decomposition:** Localization properties of localization, primary decomposition; primary ideals, uniqueness of primary decomposition, exercises based on above topics.

### Unit IV

**Integral dependence:** Integral dependence; transitivity of integral dependence, going-Up and going down theorems, exercises based on above topics.

### Unit V

**Noetherian rings:** Chain condition; Noetherian and Artinian modules, Noetherian rings; Hilbert basis theorem, irreducible ideals and primary decomposition in Noetherian rings, exercises based on above topics.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain the concept, examples and fundamental properties of Ring, ring homomorphism, ideals, quotient rings, zero-divisors, nilpotents and units, prime and maximal ideals, local rings, Nilradical and Jacobson radicals.
2. explain the concept, examples and properties of module, Module homomorphism, Sub - modules, Quotient modules, direct sum and product of modules, Finitely generated modules and Tensor product of modules.
3. explain the fundamental theorems such as Nakayama lemma.

4. explain the concept and properties of Localization and primary decomposition.
5. explain the concept and properties of Integral dependence, transitivity of integral dependence.
6. explain the some fundamental theorems such as going-Up and going down theorems, Hilbert basis theorem.
7. explain the concept of Noetherian and Artinian modules, Noetherian rings, irreducible ideals and primary decomposition in Noetherian rings.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books recommended:**

**TEXT BOOKS:**

1. **Atiyah, M. f. and Macdonald, I. G.,(1994)**, Introduction to Commutative Algebra, **Addison-Wesley Publishing Company**.

**REFERENCE BOOKS:**

1. **Eisenbud, D., (1999)**, Commutative Algebra; With a View Toward Algebraic Geometry **Springer- Verlag, New York.**,
2. **Kunz, E. (1985)**, Introduction to Commutative Algebraic Geometry, **Birkhauser**. **Reid, M. (1996)**, Undergraduate Commutative Algebraic: London Mathematical Society Student Texts, **Cambridge University Press, Cambridge.**

## SEMESTER III

**Course Title: Advanced Complex Analysis**  
**Course Code: MM-310**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to learn the advance topics of complex analysis.*

### Unit I

**Some fundamental theorems and direct analytic continuation :** Hadamard three circle theorem, Schwarz's Lemma and its consequences, doubly periodic entire functions, zeros of certain polynomials, direct analytic continuation.

### Unit II

**Some fundamental theorems and infinite sums and products:** Monodromy theorem, Poisson integral formulae, analytic continuation via reflexion, infinite sums of meromorphic functions, infinite product of complex numbers.

### Unit III

**Infinite products of analytic functions:** Infinite product of analytic functions, factorization of entire functions, the gamma functions, zeta functions, order and the genus of entire functions, open mapping and Herwitz theorem.

### Unit IV

**Univalent functions and some fundamental theorems:** Basic results on univalent functions, normal families, the Riemann mapping theorem, Biberbach conjecture, the Bloch-Landau theorem, Picard's theorem.

### Unit V

**Navlinna theory:** Jensen's formula, Navlinna theory, Navlinna's 1<sup>st</sup> and 2<sup>nd</sup> fundamental theorem, order of meromorphic functions.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain the concept of Direct analytic continuation and double periodic entire functions.
2. explain Monodromy theorem, Poisson integral formulae, open mapping and Herwitz theorem, Hadamards three circle theorem, Schwarz lemma and its various consequences.
3. Explain the concept of infinite sum of meromorphic functions and infinite product of analytic functions.
4. explain factorization of entire functions, the gamma functions, zeta functions, order and the genus of entire functions.

5. Explain the concept and basic properties of univalent functions and normal families.
6. Explain some fundamental theorems such as the Riemann mapping theorem, Biberbach conjecture, the Bloch-Landau theorem, Picard's theorem.
7. Explain the basics of Navlinna's theory with special emphasis on Narlinnas first and second fundamental theorem.
8. Explain the concept of order of a mermorphic function.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Ponnusamy, S., (1972),** Foundation of Complex Analysis, 2nd edition, **Narosa Publishing House.**
2. **Ahlofrs, L. R., (1996),** Complex Analysis, **McGraw Hill.**

**REFERENCE BOOKS:**

**Holland,A. S. B., (1973),** Introduction to the Theory of Entire Functions, **Academic Press.**

## SEMESTER III

**Course Title: Abstract Measure Theory  
and Integration**

**Course Code: MS-311**

**Credits: 4**

**Maximum Marks: 100**

**University Examination: 60**

**Sessional Assessment: 40**

**Duration of Exam: 3 hours**

### **Objectives:**

*The aim of this course to study measure theory and integration in abstract setting.*

### **Unit I**

**Abstract integration:** Measureable space, sets and function; fundamental operations on measureable functions; measure and its elementary properties; integration of simple functions; integration of positive functions

### **Unit II**

**Abstract integration and positive Borel measure:** Lebesgue monotone convergence theorem; Fatou's lemma; integration of complex functions, Lebesgue dominated convergence theorem ; role played by sets of measure zero, Riesz representation theorem (statement only); properties of Borel measure; existence of Lebesgue measure on  $\mathbb{R}$  (statement only) ; Lusin's and Vitali-Carathéodory theorems (statements only).

### **Unit III**

**$L^p$  spaces and complex Measure:** Convex functions and Jensen's inequality;  $L^p$  spaces and their completeness; approximation by continuous functions, complex measure and its total variation; positive and negative variations; absolute continuity; theorem of Lebesgue-Radon-Nikodym (statement only) and its consequences; Hahn decomposition

### **Unit IV**

**Complex measures and differentiation:** Bounded linear functionals on  $L^p$  ; Riesz representation theorem (statement only), derivatives of measures; Lebesgue points; nicely shrinking sets; fundamental theorem of calculus

### **Unit V**

**Integration on product spaces:** Measurability on cartesian product; product measures, Fubini's theorem; completion of product measure; convolutions; distributions functions

### **Course Outcomes**

After studying this course we expect a student should be able to

1. explain the concept, examples and properties of Measureable space, measureable sets, measureable functions, measures and Borel sets.
2. explain the concept, examples and properties of integral of measureable function.
3. explain some fundamental theorems such as Lebesgue monotone convergence theorem, Fatou's lemma, Lebesgue dominated convergence theorem, Riesz representation theorem, Lusin's and Vitali-Carathéodory theorems, Jensen's inequality

4. explain the concepts of  $L^p$ -space and its various features such as completeness and Bounded linear functionals on it.
5. explain the concepts of complex measure, total variation, positive and negative variations, absolute continuity and some fundamental results such as Lebsgue-Radon-Nikodym(with consequences) and Hahn decomposition theorem.
6. explain the concepts of derivatives of a measure, Lebsgue points, nicely shrinking sets.
7. explain the concepts of product measures, completion of product measure, convolutions and distributions functions.
8. Explain some fundamental theorems such as fundamental theorem of calculus, Fubni's theorem with applications.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books recommended:**

**TEXT BOOKS:**

1. **Rudin, W.,(1987)**, Real and Complex Analysis, 3<sup>rd</sup>Edition, **Tata Mcgraw-Hill Edition.**

**REFERENCE BOOKS:**

1. **Royden, H.L., (2006)**,Real Analysis, 3<sup>rd</sup> edition, **Prentice-hall of India Private Limited.**
2. **Yeh, J., (2000)**, Lectures on Real Analysis, **World Scientific.**

### SEMESTER IV

Course Code	Course Title	No. of Credits	Distribution of Marks		
			SA	UE	Total
<b>Core courses</b>					
MS-401	Dissertation/ Major Project	08	50 (D=30; V=20)	150 (D=100; V=50)	200
MS-402	Technical Communication	02	20	30	50
MS-403	Lab course on SPSS	02	25	25	50
<b>Choice based Complementary Electives</b>					
The students are required to choose any three of the following courses					
MS-404	Complex Dynamics	04	40	60	100
MS-405	Banach Algebras	04	40	60	100
MS-406	Advanced Functional Analysis	04	40	60	100
MS-407	Tensor Analysis and Riemannian Geometry	04	40	60	100
MS-408	Algebraic Topology	04	40	60	100
MS-409	Theory of Fields	04	40	60	100
MS-410	Spaces of Analytic Functions	04	40	60	100
MS-411	Algebraic Geometry	04	40	60	100
MS-412	Theory of Relativity	04	40	60	100
	<b>Total</b>	<b>24</b>	<b>210</b>	<b>390</b>	<b>600</b>

**SA: Sessional Assessment**  
**UE: University Examination**  
**D: Dissertation**  
**V: Viva-Voce**

## SEMESTER IV

**Course Title: Dissertation/Major Project**  
**Course Code: MS-401**  
**Credits: 8**

**Maximum Marks: 200**  
**University Examination: 150**  
**Sessional Assessment: 50**

### Objectives

*The objective of this course is to give a glimpse of research methods.*

Each student has to submit a project on a topic of his/her own choice under the supervision of a teacher selected as guide by the student's choice from the Departmental faculty or under the joint supervision of a teacher from the Department and an appropriate member from any other Department or industry but after the permission of the Departmental guide. The marks by internal /external examiner will be assigned on the basis of the project report submitted by the student and the viva-voce examination. The breakup for the dissertation and viva-voce marks is as follows:

	<u>Dissertation</u>	<u>viva-voce</u>	<u>Total</u>
Supervisor:	30	20	50
External Examiner:	100	50	150

### **Course Outcomes:**

After a student completes the Major project, we expect a student have understood

1. the method of searching literature, on a particular topic, form the internet.
2. the various potential areas of research, in a particular field, that can lead to a research degree(M. Phil/ Ph. D).
3. various ethics of good research.
4. how to read a research paper and present it in his / her own words.
5. the use of various concepts from different courses for studying a research paper.
6. how to employ the skills learned through different courses to simplify complicated situations.
7. the value of teamwork.



## SEMESTER IV

**Course Title: Technical Communication**  
**Course Code: MS-402**  
**Credits: 2**

**Maximum Marks: 50**  
**University Examination: 30**  
**Sessional Assessment: 20**  
**Duration of Exam: 2 hours**

### **Objectives:**

*The objective of teaching English to the students of Mathematics is to make them acquainted with English language which is now considered a global language. Acquaintance with English language will increase their prospects of employability and increase their communication skill as well.*

### **Unit I**

**Communication-I:** Scope and importance of communication; barriers to communication; verbal, non-verbal, oral and written communication; techniques to improve communication; presentation skills - effective use of presentation software and overhead, practical sessions.

### **Unit II**

**Communication-II:** Parts of speech; words frequently misspelt; formation of words; tenses; one word substitutions; use of preposition; précis writing; narration; change of voices; paragraph writing; punctuation.

### **Unit III**

**Writing skills, group discussion and interview:** Rules of good writing; principles of letter writing - structure and layout; curriculum vitae; letter of acceptance; letter of resignation; application / letters with bio-data; notice; agenda; minutes; *group discussion* - definition, methodology, helpful expression and evaluation with practical sessions; *interview* - types of interview and interview skills with practical session.

### **Course Outcomes:**

After completing this course we expect a student should

1. be able to explain the importance of good communication skills in verbal, non-verbal, oral and written communication.
2. be able to explain the techniques to improve communication and presentation skills.
3. be able to write reports etc in a precise and correct way.
4. be able to explain the basic principles of good writing.
5. be able to explain the method of presenting one's curriculum vitae.
6. be able to write various official and unofficial letters, notices, agendas, minutes of meetings etc.
7. know how to behave in a group discussion with better expressions.
8. know how to behave in an interview with better expressions.

### **Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 06 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 06 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 08 marks.*

### **Books Recommended:**

#### **TEXT BOOKS:**

1. **Balasubramanian, T., (1981)**,A Textbook of English Phonetics for Indian students, **MacMillan India Ltd.**
2. **Eastwood, J., (1999)**,Oxford Practice Grammar, **Oxford University Press.**
3. **Jones, L.,(1998)**, Cambridge Advanced English, **Cambridge University Press.**

#### **REFERENCE BOOKS:**

1. **Lesikar, R. V. and Pettir, Jr., (2004)**, Business Communication Theory and Applications, 6<sup>th</sup> edition, **A. I. T. B. S, New Delhi.**
2. **Thakar, P. K., Desai, S.D. and Purani, J. J., (1998)**,Developing English Skills, **Oxford University Press.**

## SEMESTER IV

**Course Title: Lab course on SPSS**  
**Course Code: MS-403**  
**Credits: 0**

**Maximum Marks: 50**  
**University Examination: 25**  
**Sessional Assessment: 25**  
**Duration of Exam: 3 hours**

### **Objectives:**

*The objective of this course is to introduce the basic working of the SPSS software.*

Each student is required to maintain a practical record book .

The course carries 50 marks. Two practical tests, one Internal and one External, are to be conducted, each carrying 25 marks. The student has to pass both internal and external practical test separately scoring a minimum of 10 marks for each test.

## SEMESTER IV

**Course Title: Complex Dynamics**  
**Course Code: MS-404**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course is to study fundamentals of complex dynamics –iterations of functions in a plane; Fatou and Julia sets.*

### Unit I

**Iterations of functions and various metrics on  $\mathbb{C}^\infty$ :** Iteration of a Mobius transformation; attracting, repelling and indifferent fixed points; iterations of  $R(z) = z^2, z^2+c, z+1/z$ ; the extended complex plane; chordal metric; spherical metric; relation between chordal metric and spherical metric; rational maps; Lipschitz condition.

### Unit II

**Conjugacy classes of rational maps:** Conjugacy classes of rational maps; valency of a function; fixed points; critical points; Riemann Hurwitz relation; equicontinuous functions; normality sets; Fatou sets and Julia sets.

### Unit III

**Julia sets:** Completely invariant sets; normal families and equicontinuity; properties of Julia sets; exceptional points; backward orbit; minimal property of Julia sets; Julia sets of commuting rational functions.

### Unit IV

**Fatou sets:** Structure of Fatou set; topology of the sphere; completely invariant components of the Fatou set; the Euler characteristic; Riemann Hurwitz formula for covering maps.

### Unit V

**Components of Fatou and Julia sets:** Various maps between components of the Fatou sets; number of components of Fatou sets; various components of Julia sets.

### Course Outcomes:

After completing this course we expect a student should be able to

1. explain the concepts of repelling points, attracting points and indifferent fixed points.
2. explain the concept of extended complex plane, chordal metric, spherical metric and relationship between chordal and spherical metrics.
3. explain the concepts of conjugacy class of a rational map, valency of a function, completely invariant sets, normal families and equicontinuous family of functions.

4. explain the minimal property of Julia sets and Julia sets of commuting rational functions.
5. explain the concepts of Fatou sets, Julia sets and relationship between them.
6. explain the topology of the sphere, the Euler characteristic and Riemann Hurwitz formula for covering maps.
7. explain the maps between components of Fatou and Julia sets.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Beardon, A. F. n,(1991)**, Iteration of Rational Functions, **Springer Verlag, New York.**
2. **Carleson, L. and Gamelin, T . W., (1993)**, Complex Dynamics, **Springer Verlag.**
3. **Morosawa, S., Nishimura, Y., Taniguchi, M., Ueda, T., (2000)**, Holomorphic Dynamics, **Cambridge University Press.**

**REFERENCE BOOKS:**

1. **Hua, X. H., Yang, C. C., (2000)**, Dynamics of Transcendental Functions, **Gordan and Breach Science.** **Livi, R., Nadal, J. P. and Packard, N., (1993)**, Complex Dynamics, **Nova Science Publication, Inc.**

## SEMESTER IV

**Course Title: Banach Algebras**  
**Course Code: MS-405**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study Banach algebra and its spectral theory.*

### Unit I

**Banach Algebra and its spectral properties:** Definition, examples and elementary properties of Banach Algebra; ideals in a Banach algebra; properties of set of invertible elements of a Banach algebra; properties of maximal ideals of a Banach algebra; quotient space of a Banach algebra; spectral of an element of a Banach algebra; formula for calculating spectral radius.

### Unit II

**Spectral and Riesz functional calculus:** Riesz functional calculus and its uniqueness; spectral mapping theorem; dependence of the spectral on the algebra; spectral of a linear operator; approximate point spectral of a linear operator.

### Unit III

**Abelian Banach algebra and C\* algebra:** Gelfand - Mazur theorem; maximal ideal space of a Banach algebra and its properties; Gelfand transforms and its properties; radical of a Banach algebra; definition, examples and elementary properties of C\* algebra; Abelian C\* - algebra and the functional calculus in C\* - algebra.

### Unit IV

**C\* - algebra – I :** Hermitian elements; positive elements in C\* - algebra; space of positive elements and their properties; polar decomposition; ideals and quotients in C\*-algebra; representation of a c\* - algebras; cyclic representation; state of a c\*-algebra; Gelfand – Naimark – Segal construction.

### Unit V

**C\*-algebra – II:** Spectral measures; WOT; SOT; spectral theorem; topologies on  $B(H)$ ; commuting operators; double commutant theorem; Fuglede – Putnam theorem; Abelian Van Neumann algebras.

### Course Outcomes:

After completing this course we expect a student should be able to

1. explain the concept, examples and properties of Banach Algebra, quotient space of a Banach algebra and the set of invertible elements of a Banach algebra.
2. explain the concept of ideals and maximal ideas of a Banach algebra.
3. explain the concept of spectrum of an element of a Banach algebra and formula for calculating spectral radius.

4. explain Riesz functional calculus and its uniqueness, spectral mapping theorem and dependence of the spectral on the algebra.
5. explain Gelfand - Mazur theorem, Gelfand transforms and its properties, radical of a Banach algebra and maximal ideal space of a Banach algebra with its properties.
6. explain the concept and elementary properties of  $C^*$  algebra, Abelian  $C^*$  - algebra, functional calculus in  $C^*$  - algebra, positive elements in  $C^*$  - algebra and their space with properties.
7. explain the concept of representation of a  $c^*$  - algebra, state of a  $c^*$ -algebra, Gelfand – Naimark – Segal construction and Abelian Van Neumann algebra.
8. explain some fundamental theorems such as double commutant theorem and Fuglede – Putnam theorem.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Conway, J. B., (2008)**, A Course in Functional Analysis, 2nd edition, **Springer**.

**REFERENCE BOOKS:**

1. **Douglas, R. G., (2008)**, Banach Algebra Techniques in Operator Theory, 2<sup>nd</sup> edition, **Springer**.

## SEMESTER IV

**Course Title: Advanced Functional Analysis**  
**Course Code: MS-406**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study advance topics of functional analysis.*

### Unit I

**Topological vector spaces (TVS):** Definition and examples of topological vector spaces; convex and absorbing sets; translation and multiplication operators; local base in a TVS; types of TVS; separation properties; simple properties of closure and interior in TVS.

### Unit II

**Linear transformations:** Continuity of linear mappings; finite dimensional spaces; relation between LCTVS and its dimension; metrization; relation between F-space and closed subspace of a TVS; bounded linear transformations; semi norm and local convexity; properties of semi norm sets; Minkowski's functional and its properties.

### Unit III

**Fundamentals theorems and special spaces:** Necessary and sufficient condition for a TVS to be normable; quotient spaces of a TVS; semi norm and quotient spaces; the spaces  $C(\Omega)$ ,  $H(\Omega)$ ;  $C^\infty(\Omega)$  and  $Q_k$ ,  $L^p(0 < p < \infty)$ ; equicontinuity; Banach – Steinhaus theorem; continuity of limits of sequences of continuous linear mappings; open mapping theorem and its corollaries.

### Unit IV

**Some fundamental theorems:** Closed graph theorem; bilinear mappings; dual space; Hahn-Banach separation theorem and its various corollaries; the weak topology of a TVS; the weak\* topology of dual space of a TVS; Banach- Alaogule theorem and its applications.

### Unit V

**Convexity:** Convex Hull of a subset of a TVS and its properties; extreme points; the Krein- Milman's theorem; Milman's theorem; polar; bipolar theorem; Barelled and Bornological spaces; semi reflexive and reflexive topological vector spaces.

### Course Outcomes:

After completing this course we expect a student should be able to

1. explain the concept and examples of topological vector spaces(TVS), convex and absorbing sets, local base in a TVS and Locally convex TVS with its relation with dimension of the space.
2. explain the separation properties in a TVS and the concept of closure and interior in a TVS.



3. explain the concept and properties of continuity of linear mappings and relationship between F-space and closed subspace of a TVS.
4. explain the concept of semi norm, its various properties and MinKowski's functional.
5. explain some Fundamental theorems such as Banach – Steinhaus theorem, open mapping theorem (with consequences), Closed graph theorem, Hahn-Banach separation theorem (with corollaries), Banach- Alaogule theorem (with applications), the Krein- Milman's theorem, Milman's theorem and bipolar theorem.
6. explain the necessary and sufficient condition for a TVS to be normable and quotient spaces of a TVS.
7. explain the spaces  $C(\Omega)$  ,  $H(\Omega)$ ;  $C^\infty(\Omega)$  and  $Q_k$  ,  $L^p(0 < p < 1)$  and the continuity of limit of sequence of continuous linear mappings.
8. explain the concept of bilinear mappings, the weak and weak\* topology, Convex Hull (with properties), extreme points, Barelled and Bornological spaces, semi reflexive and reflexive topological vector spaces.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Rudin, W., (1973), Functional Analysis, Tata Mcgraw Hill.**

**REFERENCE BOOKS:**

1. **Schwartz, L., (1975), Functional Analysis, Courant Institute of Mathematical Sciences.**
2. **Treves, F. (1967), Topological Vector spaces, Distributions and Kernels Academics Press.**
3. **Kothe, G. (1976), Topological Vector Spaces-II, Springer Verlag, New York.**
4. **Larsen, R., (1972), Functional Analysis, Marcel Dekker.**

## SMESTER IV

**Course Title: Tensor Analysis and Riemannian Geometry**  
**Course Code: MS-407**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study fundamental ideas of tensors and their various types in detail.*

### Unit I

**Tensors:** Idea of differentiable manifolds with  $n$ -dimensions; space of  $n$  dimensions, subspaces; transformation of coordinates; scalar; contravariant (tangent) and covariant(cotangent) vectors; scalar product of two vectors; tensor space of rank more than one contravariant and covariant tensors; symmetric and skew-symmetric tensors; addition and multiplication of tensors; contraction; composition of tensors; quotient law; reciprocal symmetric tensors of the second order.

### Unit II

**Tensors and vectors:** Riemannian space; fundamental tensor; length of a curve; magnitude of a vector; associated covariant and contravariant vectors; inclination of two vectors, orthogonal vectors; coordinate hypersurfaces; coordinate curves; field of normals to a hypersurface; principle directions for a symmetric covariant tensor of the second order; Euclidean space of  $n$  dimensions.

### Unit III

**Derivative of a vector and tensor:** Levi-Civita tensors; Christoffel symbols and second derivatives; need for covariant derivative; parallel transformations; covariant derivative of a contravariant and covariant vector; curl of a vector and its derivative; covariant differentiation of a tensor; divergence of a vector.

### Unit IV

**Geodesic:** Gaussian curvature; Riemann curvature tensor; geodesics; differential equations of geodesics; geodesic coordinates; geodesic deviation; Riemannian coordinates; geodesic in Euclidean space; straight lines.

### Unit V

**Tensor and curvature:** Parallel transport along an extended curve; curvature tensor; Bianchi identities; Ricci tensor; scalar curvature; killing vector field; space-time symmetries (homogeneity and isotropy); space time of constant curvature; conformal transformations.

### Course Outcomes:

After completing this course we expect a student should be able to

1. explain the concept of a tensor (with various operations such as addition, multiplication, composition) contravariant and covariant tensors, symmetric and skew-symmetric tensors, Levi-Civita tensors, Christoffel symbols,

2. explain the idea of differentiable manifolds and contravariant (tangent) and covariant(cotangent) vectors.
3. explain the Riemannian space, coordinate hypersurfaces and field of normals to a hypersurface.
4. explain the principle directions for a symmetric covariant tensor of the second order
5. explain the covariant derivative of a contravariant and covariant vector and curl of a vector with its derivative.
6. explain the covariant differentiation of a tensor and divergence of a vector.
7. explain Gaussian curvature, Riemann curvature tensor, geodesics and its differential equations and coordinates.
8. explain the Ricci tensor, space-time symmetries (homogeneity and isotropy), space time of constant curvature and conformal transformations.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books recommended:**

**TEXTBOOKS:**

1. **Weatherburn, C. E., (1986)**,An Introduction to Riemannian Geometry and Tensor Calculus , **Cambridge University Press.**
2. **Narlikar, J.V., (1978)**, General Relativity and Cosmology, **The Mac-Millan Company of India Ltd.**

**REFERENCE BOOKS:**

1. **Srivastava, S. K. & Sinha, P. K., (1998)**,Aspects of Gravitational Interactions, **Nova Science publications Inc., Commack, NY.**
2. **Sokolnikoff, I. S., (1964)**, Tensor Analysis, **I. S. John Wiley & Sons, Inc.**

## SEMESTER IV

**Course Title: Algebraic Topology**  
**Course Code: MM-408**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study topology in algebraic context.*

### Unit I

**Homotopy-I:** Homotopy of paths; equivalence of path homotopy relation; product of paths and its basic properties; fundamental group of a topological space; homomorphism induced by continuous path.

### Unit II

**Homotopy-II:** Covering spaces; covering map examples; local homomorphism the fundamental group of circle; lifting of a map; lifting correspondence isomorphism between  $S^1$  and  $Z$ , retraction; non-retraction theorem; Brouwer fixed point theorem for the disc.

### Unit III

**Fundamental groups:** Deformation retracts and homotopy type; the fundamental group of  $S^n$ ; fundamental group of some surfaces; compactness of projective plane; non-commutativity of fundamental group of figure eight and double torus.

### Unit IV

**Covering spaces-I:** Equivalence of covering spaces; the general lifting lemma; relation between equivalent covering maps and conjugations of sub group; universal covering space; space without any universal covering space; existence of covering spaces; semi locally simply connected space.

### Unit V

**Covering spaces-II:** Covering transformation; group of covering transformation; regular covering map; orbit space; the fundamental theorem of algebra; Borsuk-Ulam theorem for  $S^2$  the bisection theorem.

### Course Outcomes:

After completing this course we expect a student should be able to

1. explain the concept of Homotopy of paths, their equivalence, product and various basic properties.
2. explain the concept of fundamental group of a topological space and homomorphism induced by a continuous path.
3. explain the concept of a covering space, covering map examples, local homomorphism and the fundamental group of circle.
4. explain some fundamental theorems such as non-retraction theorem and Brouwer fixed point theorem for the disc.

5. explain the concept of Deformation retracts and homotopy type and the fundamental group of  $S^n$  with its basic properties such as non commutativity of fundamental group of figure eight and double torus.
6. explain some fundamental theorems such as the general lifting lemma, the fundamental theorem of algebra, Borsuk-Ulam theorem for  $S^2$  and the bisection theorem.
7. explain equivalence of covering spaces, relation between equivalent covering maps and conjugations of sub group, existence of covering spaces and semi locally simply connected space.
8. explain the covering transformation, group of covering transformations and regular covering map.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Munkers, J.R. ,(2000),** Topology, 2<sup>nd</sup> Edition, **PHI.**

**REFERENCE BOOKS:**

1. **Greenberg, J. M. and Harper, R. J., (1981),** Algebraic Topology: A First Course, **ABP.**

## SEMESTER IV

**Course Title: Theory of Fields**  
**Course Code: MS-409**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study the advance topics algebra in field theory.*

### Unit I

**Finite and algebraic extension:** Definition and examples of field; extension fields; finite extension; transitivity of finite extension property; algebraic element; necessary and sufficient condition for an element to be algebraic in terms of dimension of the smallest field; subfield of algebraic elements; algebraic extension and transitivity of algebraic extension property; algebraic number; transcendence of  $e$ .

### Unit II

**Roots of polynomials and construction with straight edge and compass:** Roots of a polynomial over field; remainder theorem; number of a roots a polynomial in an extension field; existence of an extension of  $F$  of an irreducible polynomial over  $F$ ; splitting field; uniqueness of splitting field; constructible real numbers and their properties; impossibility of trisecting  $60^\circ$ , duplicating cube and constructing a regular septagon by straight edge and compass; derivative of a polynomial; simple extension; relation between simple extension and characteristic of a field.

### Unit III

**Galois theory:** Automorphism of a field; fixed field of a group; the group  $G(K,F)$ ; the inequality  $O(G(K,F)) \leq [K:F]$ ; field of symmetric rational function and its properties; normal extension and its relation with splitting field, Galois group of a polynomial; fundamental theorem of Galois theory.

### Unit IV-

**Solvability by radicals and Galois group over the rationals:** Solvable group; commutator subgroups; relation between solvability and commutator subgroups; homomorphic image of a solvable group; non-solvability of  $S_n$  ( $n \geq 5$ ); relation between solvability by radicals of a polynomial and solvability of the Galois group; non-solvability of polynomial of degree  $\geq 5$ ; Galois group; simple extension based on above topics.

### Unit V

**Finite fields:** Number of elements in a finite field; finite fields having same number of elements; existence of finite fields; group of non-zero elements of a field; roots of an irreducible polynomials over finite fields; nature of roots; relation between splitting field of two irreducible polynomials of same elements.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain the concept of finite extensions, algebraic elements, algebraic numbers and transcendence of  $e$ .

2. explain the concept of roots of polynomial over field, remainder theorem, irreducible polynomials, splitting field, constructible real numbers and their properties.
3. explain the relation between simple extension and characteristic of a field.
4. explain the concept of automorphism of a field, fixed field of a group and normal extension.
5. explain the concept of fundamental theorem of Galois theory, Galois group of a polynomial.
6. explain the concept of solvable group, commutator subgroup, relation between solvability and commutator subgroup.
7. explain the concept of radicals, radicals of a polynomial and solvability of polynomial of degree  $\geq 5$ .
8. explain the concept of finite field, existence of a finite field and roots of irreducible polynomials over finite fields.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Herstein, I. N., (2004),** Topics in Algebra, 2<sup>nd</sup> edition, **Wiley Student Edition.**

**REFERENCE BOOKS:**

1. **Lidl, R. and Pilz G. , (2004),** Applied Abstract Algebra, 2<sup>nd</sup> edition, **Springer.**

## SEMESTER IV

**Course Title: Spaces of Analytic Functions**  
**Course Code: MS-410**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to introduced the students by the concept of Fourier series and its applications.*

### Unit I

**Fourier series:** Review of Fourier series, Fourier transform and its properties; convolution theorem; the inversion theorem; uniqueness theorem; Plancherel's theorem; Parseval's formula.

### Unit II

**Fourier transform and harmonic functions:** Translation invariant subspaces of  $L^2$ ; the Banach algebra  $L^1$ ; complex homomorphism; the complex homomorphism of  $L^1$ ; Cauchy-Riemann equation; The Laplacian; Poisson kernel; the poisson integral of a  $L^1$  function; Harnack's theorem.

### Unit III

**Mean value property:** Mean value property; the Schwarz reflection principle; boundary behavior of Poisson integrals; Poisson integrals of measures; approach regions; maximal functions; non-tangential limits; representation theorems; Arzela-Ascoli theorem.

### Unit IV

**Hardy spaces over the unit disk:** Sub-harmonic functions, Hardy space  $H^p(U)$  in  $H^{pn}(U)$  as a Banach space, Blaschke product and its properties, Navanlinna space  $N$ , theorem of F and M Riesz, inner and outer functions factorization.

### Unit V

**Hardy spaces over the upper-half plane:** Sub-harmonic functions in the upper-half-plane, Hardy space  $H^p(\mathbb{T}^+)$  over the upper half plane, Poission integral formula; Cauchy integral formula; boundary behavior of functions in  $H^p(\mathbb{T}^+)$ ; canonical factorization  $H^p(\mathbb{T}^+)$  as a Banach space; Paley – Wiener theorem.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain the concept of Fourier series, Fourier transform(with properties) and some basic theorems such as convolution theorem, the inversion theorem, uniqueness theorem, Plancherel's theorem and Parseval's formula.
2. explain Translation invariant subspaces of  $L^2$ , the Banach algebra  $L^1$ , Poisson kernel and the poisson integral of a  $L^1$  function, the Laplacian and some basic theorems such as Cauchy-Riemann equation, Harnack's theorem.



3. Explain the concept of Mean value property, maximal functions, non-tangential limits, boundary behavior of Poisson integrals and Poisson integrals of measures.
4. explain some fundamental results such as the Schwarz reflection principle, representation theorems, Arzela–Ascoli theorem.
5. explain the concept of sub-harmonic functions, Hardy space  $H^p(U)$  and its various features such as its Banachness.
6. explain Blaschke product (with properties), Navanlinna space  $N$ , the theorem of F and M Riesz and inner and outer functions factorization.
7. explain Sub-harmonic functions in the upper-half-plane, Hardy space  $H^p(\Pi^+)$  over the upper half plane and its features.
8. explain Poisson integral formula, Cauchy integral formula, boundary behavior of functions in  $H^p(\Pi^+)$ , canonical factorization  $H^p(\Pi^+)$  as a Banach space and Paley – Wiener theorem.

**Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

**Books Recommended:**

**TEXT BOOKS:**

1. **Duren, P. L., (1970),** Theory of HP Spaces, **Academic Press.**
2. **Rudin, W., (1987),** Real and Complex Analysis, 3<sup>rd</sup> edition, **McGraw Hill Book Co.**

**REFERENCE BOOKS:**

1. **Carnett, J. B.,(1981),** Bounded Analytic Functions, **Academic Press.**
2. **Hoffman, K., (2009),**BanachSpaces of Analytic Functions, **Prentice Hall Engle wook Cliffs, New Jersay.**

## SEMESTER IV

**Course Title: Algebraic Geometry**  
**Course Code: MS-411**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 60**  
**Sessional Assessment: 40**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to introduce the students by the concept of algebraic geometry and its applications*

### Unit I

**Rational maps:** Introduction; affine varieties, Hilbert's Null stellensatz, polynomial function and maps; rational functions and maps.

### Unit II

**Smoothness, singularity and dimension:** Projective space; projective varieties; rational functions and morphisms; smooth points and dimension, smooth and singular points, algebraic characterizations of the dimension of a variety.

### Unit III

**Plane curves:** Plane cubic curves, plane curves, intersection multiplicity, classification of smooth cubics, the group structure of an elliptic curve.

### Unit IV

**Cubic surfaces:** Cubic surfaces, the existence of lines on a cubic, configuration of the 27 lines, rationality of cubics.

### Unit V

**Theory of curves:** Introduction to the theory of curves, divisors on curves, the degree of a principal divisor, Bezout's theorem, linear system on curves, projective embeddings of curves.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain rational functions and maps, affine varieties and their properties.
2. explain Projective space and projective varieties and algebraic characterizations of the dimension of a variety.
3. explain the Plane cubic curves and intersection, multiplicity, classification of smooth cubics.
4. explain the group structure of an elliptic curve.
5. explain Cubic surfaces and the existence of lines on a cubic.
6. explain configuration of the 27 lines and the rationality of cubics.
7. explain divisors on curves and the degree of a principal divisor
8. explain the Bezout's theorem and projective embeddings of curves.

### **Note for Paper Setting:**

*The question paper will be divided into two sections. **Section A** will be compulsory and will contain 10 very short answer type questions eliciting answers not exceeding 20 words/ multiple choice questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 long answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 10 marks.*

### **Books Recommended:**

#### **TEXTBOOKS:**

1. **Hulek, K. (translated by H. Verrill), (2003),** Elementary Algebraic Geometry–Student Mathematical Library, vol 20, **American Mathematical Society.**

#### **REFERENCE BOOKS:**

1. **Hartshorne, R., (1977),** Algebraic Geometry, **Springer Verlag.**
2. **Harris, J., (1992),**Algebraic Geometry: A First Course, **Springer Verlag.**
3. Elliptic Curves, Notes on NBHM Instructional Conference held at TIFR,(1991),**Mumbai.**

## SEMESTER IV

**Course Title: Theory of Relativity**  
**Course Code: MS-412**  
**Credits: 4**

**Maximum Marks: 100**  
**University Examination: 70**  
**Sessional Assessment: 30**  
**Duration of Exam: 3 hours**

### Objectives:

*The aim of this course to study the theory of relativity and its applications.*

### Unit I

**The special theory of relativity:** inertial frames of reference; postulates of the special theory of relativity; Lorentz transformations; length contraction; time dilation; variation of mass; composition of velocities; relativistic mechanics; world events, world regions and light cone; Minkowski space-time; equivalence of mass and energy.

### Unit II

**Energy-momentum tensors:** the action principle; the electromagnetic theory; energy-momentum tensors (general); energy-momentum tensors (special cases); conservation laws.

### Unit III

**General theory of Relativity:** Introduction; principle of covariance; principle of equivalence; derivation of Einstein's equation; Newtonian approximation of Einstein's equations.

### Unit IV

**Solution of Einstein's equation and tests of general relativity:** Schwarzschild solution; particle and photon orbits in Schwarzschild space-time; gravitational red shift; planetary motion; bending of light; radar echo delay.

### Unit V

**Brans-Dicke theory:** Scalar tensor theory and higher derivative gravity; Kaluza-Klein theory.

### Course Outcomes

After studying this course we expect a student should be able to

1. explain postulates of the special theory of relativity
2. explain the concept of inertial frames of reference, Lorentz transformations, length contraction, time dilation, variation of mass, composition of velocities,
3. explain Minkowski space-time concept and equivalence of mass and energy, the idea of action principle and energy-momentum tensors (general and special cases).
4. explain the conservation laws and general theory of relativity.
5. explain various principles such as principle of covariance and principle of equivalence.

6. explain the Einstein's equation and Newtonian approximation of Einstein's equations.
7. explain the concepts of Schwarz's child solution, particle and photon orbits in Schwarzschild space-time.
8. explain Scalar tensor theory, higher derivative gravity and Kaluza-Klein theory.

**Note for Paper Setting:**

*The question paper will be divided into three sections. **Section A** will be compulsory and will contain 15 very short answer type questions eliciting answers not exceeding 20 words/ multiple choices questions/ fill in the blanks, each carrying one mark equally distributed from all units. **Section B** will contain 10 short answer type questions, two from each unit and the candidate will be required to answer one from each unit. Each question carries 05 marks. **Section C** will contain five long answer type questions, one from each unit and the student will attempt any three questions. Each question carries 10 marks.*

**Book recommended:**

**TEXTBOOKS:**

1. **Narlikar, J.V., (1988),** General Relativity & Cosmology, 2nd edition, **Macmillan Co. of India Limited.**
2. **Pathria, R.K.,(1994),** The Theory of Relativity, 2nd edition, **Hindustan Publishing Co. Delhi.**

**REFERENCE BOOKS:**

1. **Srivastava, S. K. and Sinha, K. P., (1998),**Aspects of Gravitational Interactions, NovaScience Publishers Inc. Commack, New York.
2. **Rindler,W., (1977),** Essential Relativity, **Springer-Verlag.**
3. **Wald, R.M., (1984),**General Relativity, **University of Chicago Press.**